Introduction

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Contents

1 Background

T(opological) G(eometro)D(ynamics) is one of the many attempts to find a unified description of basic interactions. The development of the basic ideas of TGD to a relatively stable form took time of about half decade [16]. The great challenge is to construct a mathematical theory around these physically very attractive ideas and I have devoted the last twenty-three years for the realization of this dream and this has resulted in seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15].

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness have been for last decade of the second millenium the basic three strongly interacting threads in the tapestry of quantum TGD.

For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. The work with Riemann hypothesis made time ripe for realization that the notion of infinite primes could provide, not only a reformulation, but a deep generalization of quantum TGD. This led to a thorough and extremely fruitful revision of the basic views about what the final form and physical content of quantum TGD might be.

The fifth thread came with the realization that by quantum classical correspondence TGD predicts an infinite hierarchy of macroscopic quantum systems with increasing sizes, that it is not at all clear whether standard quantum mechanics can accommodate this hierarchy, and that a dynamical quantized Planck constant might be necessary and certainly possible in TGD framework. The identification of hierarchy of Planck constants whose values TGD "predicts" in terms of dark matter hierarchy would be natural. This also led to a solution of a long standing puzzle: what is the proper interpretation of the predicted fractal hierarchy of long ranged classical electro-weak and color gauge fields. Quantum classical correspondences allows only single answer: there is infinite hierarchy of p-adically scaled up variants of standard model physics and for each of them also dark hierarchy. Thus TGD Universe would be fractal in very abstract and deep sense.

TGD forces the generalization of physics to a quantum theory of consciousness, and represent TGD as a generalized number theory vision leads naturally to the emergence of p-adic physics as physics of cognitive representations. The seven online books [1, 2, 4, 5, 3, 6, 7] about TGD and eight online books about TGD inspired theory of consciousness and of quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended to the interested reader.

2 Basic Ideas of TGD

The basic physical picture behind TGD was formed as a fusion of two rather disparate approaches: namely TGD is as a Poincare invariant theory of gravitation and TGD as a generalization of the old-fashioned string model.

2.1 TGD as a Poincare invariant theory of gravitation

The first approach was born as an attempt to construct a Poincare invariant theory of gravitation. Space-time, rather than being an abstract manifold endowed with a pseudo-Riemannian structure, is regarded as a surface in the 8-dimensional space $H = M^4_+ \times CP_2$, where M^4_+ denotes the interior of the future light cone of the Minkowski space (to be referred as light cone in the sequel) and $CP_2 = SU(3)/U(2)$ is the complex projective space of two complex dimensions [17, 18, 19, 20]. The identification of the space-time as a submanifold [21, 22] of $M^4 \times CP_2$ leads to an exact Poincare invariance and solves the conceptual difficulties related to the definition of the energy-momentum in General Relativity [Misner-Thorne-Wheeler, Logunov *et al*]. The actual choice $H = M^4_+ \times CP_2$

implies the breaking of the Poincare invariance in the cosmological scales but only at the quantum level. It soon however turned out that submanifold geometry, being considerably richer in structure than the abstract manifold geometry, leads to a geometrization of all basic interactions. First, the geometrization of the elementary particle quantum numbers is achieved. The geometry of CP_2 explains electro-weak and color quantum numbers. The different H-chiralities of H-spinors correspond to the conserved baryon and lepton numbers. Secondly, the geometrization of the field concept results. The projections of the CP_2 spinor connection, Killing vector fields of CP_2 and of H-metric to four-surface define classical electro-weak, color gauge fields and metric in X^4.

2.2 TGD as a generalization of the hadronic string model

The second approach was based on the generalization of the mesonic string model describing mesons as strings with quarks attached to the ends of the string. In the 3-dimensional generalization 3-surfaces correspond to free particles and the boundaries of the 3- surface correspond to partons in the sense that the quantum numbers of the elementary particles reside on the boundaries. Various boundary topologies (number of handles) correspond to various fermion families so that one obtains an explanation for the known elementary particle quantum numbers. This approach leads also to a natural topological description of the particle reactions as topology changes: for instance, two-particle decay corresponds to a decay of a 3-surface to two disjoint 3-surfaces.

2.3 Fusion of the two approaches via a generalization of the space-time concept

The problem is that the two approaches seem to be mutually exclusive since the orbit of a particle like 3-surface defines 4-dimensional surface, which differs drastically from the topologically trivial macroscopic space-time of General Relativity. The unification of these approaches forces a considerable generalization of the conventional space-time concept. First, the topologically trivial 3-space of General Relativity is replaced with a "topological condensate" containing matter as particle like 3-surfaces "glued" to the topologically trivial background 3-space by connected sum operation. Secondly, the assumption about connectedness of the 3-space is given up. Besides the "topological condensate" there is "vapor phase" that is a "gas" of particle like 3-surfaces (counterpart of the "baby universies" of GRT) and the nonconservation of energy in GRT corresponds to the transfer of energy between the topological condensate and vapor phase.

3 The five threads in the development of quantum TGD

The development of TGD has involved four strongly interacting threads: physics as infinite-dimensional geometry; p-adic physics; TGD inspired theory of consciousness and TGD as a generalized number theory. In the following these five threads are briefly described.

3.1 Quantum TGD as configuration space spinor geometry

A turning point in the attempts to formulate a mathematical theory was reached after seven years from the birth of TGD. The great insight was "Do not quantize". The basic ingredients to the new approach have served as the basic philosophy for the attempt to construct Quantum TGD since then and are the following ones:

a) Quantum theory for extended particles is free(!), classical(!) field theory for a generalized Schrödinger amplitude in the configuration space CH consisting of all possible 3-surfaces in H. "All possible" means that surfaces with arbitrary many disjoint components and with arbitrary internal topology and also singular surfaces topologically intermediate between two different manifold topologies are included. Particle reactions are identified as topology changes [23, 24, 25]. For instance, the decay of a 3-surface to two 3-surfaces corresponds to the decay $A \rightarrow B + C$. Classically this corresponds to a path of configuration space leading from 1-particle sector to 2-particle sector. At quantum level this corresponds to the dispersion of the generalized Schrödinger amplitude localized to 1-particle sector to two-particle sector. All coupling constants should result as predictions of the theory since no nonlinearities are introduced.

b) Configuration space is endowed with the metric and spinor structure so that one can define various metric related differential operators, say Dirac operator, appearing in the field equations of the theory.

3.2 p-Adic TGD

The p-adic thread emerged for roughly ten years ago as a dim hunch that p-adic numbers might be important for TGD. Experimentation with p-adic numbers led to the notion of canonical identification mapping reals to p-adics and vice versa. The breakthrough came with the successful p-adic mass calculations using p-adic thermodynamics for Super-Virasoro representations with the super-Kac-Moody algebra associated with a Lie-group containing standard model gauge group. Although the details of the calculations have varied from year to year, it was clear that p-adic physics reduces not only the ratio of proton and Planck mass, the great mystery number of physics, but all elementary particle mass scales, to number theory if one assumes that primes near prime powers of two are in a physically favored position. Why this is the case, became one of the key puzzless and led to a number of arguments with a common gist: evolution is present already at the elementary particle level and the primes allowed by the p-adic length scale hypothesis are the fittest ones.

It became very soon clear that p-adic topology is not something emerging in Planck length scale as often believed, but that there is an infinite hierarchy of p-adic physics characterized by p-adic length scales varying to even cosmological length scales. The idea about the connection of p-adics with cognition motivated already the first attempts to understand the role of the p-adics and inspired 'Universe as Computer' vision but time was not ripe to develop this idea to anything concrete (p-adic numbers are however in a central role in TGD inspired theory of consciousness). It became however obvious that the p-adic length scale hierarchy somehow corresponds to a hierarchy of intelligences and that p-adic prime serves as a kind of intelligence quotient. Ironically, the almost obvious idea about p-adic regions as cognitive regions of space-time providing cognitive representations for real regions had to wait for almost a decade for the access into my consciousness.

There were many interpretational and technical questions crying for a definite answer. What is the relationship of p-adic non-determinism to the classical non-determinism of the basic field equations of TGD? Are the p-adic space-time region genuinely p-adic or does p-adic topology only serve as an effective topology? If p-adic physics is direct image of real physics, how the mapping relating them is constructed so that it respects various symmetries? Is the basic physics p-adic or real (also real TGD seems to be free of divergences) or both? If it is both, how should one glue the physics in different number field together to get *The Physics*? Should one perform p-adicization also at the level of the configuration space of 3-surfaces? Certainly the p-adicization at the level of super-conformal representation is necessary for the p-adic mass calculations. Perhaps the most basic and most irritating technical problem was how to precisely define p-adic definite integral which is a crucial element of any variational principle based formulation of the field equations. Here the frustration was not due to the lack of solution but due to the too large number of solutions to the problem, a clear symptom for the sad fact that clever inventions rather than real discoveries might be in question.

Despite these frustrating uncertainties, the number of the applications of the poorly defined p-adic physics growed steadily and the applications turned out to be relatively stable so that it was clear that the solution to these problems must exist. It became only gradually clear that the solution of the problems might require going down to a deeper level than that represented by reals and p-adics.

3.3 TGD as a generalization of physics to a theory consciousness

General coordinate invariance forces the identification of quantum jump as quantum jump between entire deterministic quantum histories rather than time=constant snapshots of single history. The new view about quantum jump forces a generalization of quantum measurement theory such that observer becomes part of the physical system. Thus a general theory of consciousness is unavoidable outcome. This theory is developed in detail in the books [10, 8, 9, 13, 11, 12, 14, 15].

3.3.1 Quantum jump as a moment of consciousness

The identification of quantum jump between deterministic quantum histories (configuration space spinor fields) as a moment of consciousness defines microscopic theory of consciousness. Quantum jump involves the steps

$$\Psi_i \rightarrow U\Psi_i \rightarrow \Psi_f \quad ,$$

where U is informational "time development" operator, which is unitary like the S-matrix characterizing the unitary time evolution of quantum mechanics. U is however only formally analogous to Schrödinger time evolution of infinite duration although there is *no* real time evolution involved. It is not however clear whether one should regard U-matrix and S-matrix as two different things or not: U-matrix is a completely universal object characterizing the dynamics of evolution by self-organization whereas S-matrix is a highly context dependent concept in wave mechanics and in quantum field theories where it at least formally represents unitary time translation operator at the limit of an infinitely long interaction time. The S-matrix understood in the spirit of superstring models is however something very different and could correspond to U-matrix.

The requirement that quantum jump corresponds to a measurement in the sense of quantum field theories implies that each quantum jump involves localization in zero modes which parameterize also the possible choices of the quantization axes. Thus the selection of the quantization axes performed by the Cartesian outsider becomes now a part of quantum theory. Together these requirements imply that the final states of quantum jump correspond to quantum superpositions of space-time surfaces which are macroscopically equivalent. Hence the world of conscious experience looks classical. At least formally quantum jump can be interpreted also as a quantum computation in which matrix U represents unitary quantum computation which is however not identifiable as unitary translation in time direction and cannot be 'engineered'.

3.3.2 The notion of self

The concept of self is absolutely essential for the understanding of the macroscopic and macro-temporal aspects of consciousness. Self corresponds to a subsystem able to remain un-entangled under the sequential informational 'time evolutions' U. Exactly vanishing entanglement is practically impossible in ordinary quantum mechanics and it might be that 'vanishing entanglement' in the condition for self-property should be replaced with 'subcritical entanglement'. On the other hand, if space-time decomposes into p-adic and real regions, and if entanglement between regions representing physics in different number fields vanishes, space-time indeed decomposes into selves in a natural manner.

It is assumed that the experiences of the self after the last 'wake-up' sum up to single average experience. This means that subjective memory is identifiable as conscious, immediate short term memory. Selves form an infinite hierarchy with the entire Universe at the top. Self can be also interpreted as mental images: our mental images are selves having mental images and also we represent mental images of a higher level self. A natural hypothesis is that self S experiences the experiences of its subselves as kind of abstracted experience: the experiences of subselves S_i are not experienced as such but represent kind of averages $\langle S_{ij} \rangle$ of sub-subselves S_{ij}. Entanglement between selves, most naturally realized by the formation of join along boundaries bonds between cognitive or material space-time sheets, provides a possible a mechanism for the fusion of selves to larger selves (for instance, the fusion of the mental images representing separate right and left visual fields to single visual field) and forms wholes from parts at the level of mental images.

3.3.3 Relationship to quantum measurement theory

The third basic element relates TGD inspired theory of consciousness to quantum measurement theory. The assumption that localization occurs in zero modes in each quantum jump implies that the world of conscious experience looks classical. It also implies the state function reduction of the standard quantum measurement theory as the following arguments demonstrate (it took incredibly long time to realize this almost obvious fact!).

a) The standard quantum measurement theory a la von Neumann involves the interaction of brain with the measurement apparatus. If this interaction corresponds to entanglement between microscopic degrees of freedom m with the macroscopic effectively classical degrees of freedom M characterizing the reading of the measurement apparatus coded to brain state, then the reduction of this entanglement in quantum jump reproduces standard quantum measurement theory provide the unitary time evolution operator U acts as flow in zero mode degrees of freedom and correlates completely some orthonormal basis of configuration space spinor fields in non-zero modes with the values of the zero modes. The flow property guarantees that the localization is consistent with unitarity: it also means 1-1 mapping of quantum state basis to classical variables (say, spin direction of the electron to its orbit in the external magnetic field).

b) Since zero modes represent classical information about the geometry of space-time surface (shape, size, classical Kähler field,...), they have interpretation as effectively classical degrees of freedom and are the TGD counterpart of the degrees of freedom M representing the reading of the measurement apparatus. The entanglement between quantum fluctuating non-zero modes and zero modes is the TGD counterpart for the $m - M$ entanglement. Therefore the localization in zero modes is equivalent with a quantum jump leading to a final state where the measurement apparatus gives a definite reading.

This simple prediction is of utmost theoretical importance since the black box of the quantum measurement theory is reduced to a fundamental quantum theory. This reduction is implied by the replacement of the notion of a point like particle with particle as a 3-surface. Also the infinite-dimensionality of the zero mode sector of the configuration space of 3-surfaces is absolutely essential. Therefore the reduction is a triumph for quantum TGD and favors TGD against string models.

Standard quantum measurement theory involves also the notion of state preparation which reduces to the notion of self measurement. Each localization in zero modes is followed by a cascade of self measurements leading to a product state. This process is obviously equivalent with the state preparation process. Self measurement is governed by the so called Negentropy Maximization Principle (NMP) stating that the information content of conscious experience is maximized. In the self measurement the density matrix of some subsystem of a given self localized in zero modes (after ordinary quantum measurement) is measured. The self measurement takes place for that subsystem of self for which the reduction of the entanglement entropy is maximal in the measurement. In p-adic context NMP can be regarded as the variational principle defining the dynamics of cognition. In real context self measurement could be seen as a repair mechanism allowing the system to fight against quantum thermalization by reducing the entanglement for the subsystem for which it is largest (fill the largest hole first in a leaking boat).

3.3.4 Selves self-organize

The fourth basic element is quantum theory of self-organization based on the identification of quantum jump as the basic step of self-organization [I1]. Quantum entanglement gives rise to the generation of long range order and the emergence of longer p-adic length scales corresponds to the emergence of larger and larger coherent dynamical units and generation of a slaving hierarchy. Energy (and quantum entanglement) feed implying entropy feed is a necessary prerequisite for quantum self-organization. Zero modes represent fundamental order parameters and localization in zero modes implies that the sequence of quantum jumps can be regarded as hopping in the zero modes so that Haken's classical theory of self organization applies almost as such. Spin glass analogy is a further important element: self-organization of self leads to some characteristic pattern selected by dissipation as some valley of the "energy" landscape.

Dissipation can be regarded as the ultimate Darwinian selector of both memes and genes. The mathematically ugly irreversible dissipative dynamics obtained by adding phenomenological dissipation terms to the reversible fundamental dynamical equations derivable from an action principle can be understood as a phenomenological description replacing in a well defined sense the series of reversible quantum histories with its envelope.

3.3.5 Classical non-determinism of Kähler action

The fifth basic element are the concepts of association sequence and cognitive space-time sheet. The huge vacuum degeneracy of the Kähler action suggests strongly that the absolute minimum space-time is not always unique. For instance, a sequence of bifurcations can occur so that a given space-time branch can be fixed only by selecting a finite number of 3-surfaces with time like(!) separations on the orbit of 3-surface. Quantum classical correspondence suggest an alternative formulation. Space-time surface decomposes into maximal deterministic regions and their temporal sequences have interpretation a space-time correlate for a sequence of quantum states defined by the initial (or final) states of quantum jumps. This is consistent with the fact that the variational principle selects preferred extremals of Kähler action as generalized Bohr orbits.

In the case that non-determinism is located to a finite time interval and is microscopic, this sequence of 3-surfaces has interpretation as a simulation of a classical history, a geometric correlate for contents of consciousness. When non-determinism has long lasting and macroscopic effect one can identify it as volitional non-determinism associated with our choices. Association sequences relate closely with the cognitive space-time sheets defined as space-time sheets having finite time duration and psychological time can be identified as a temporal center of mass coordinate of the cognitive space-time sheet. The gradual drift of the cognitive space-time sheets to the direction of future force by the geometry of the future light cone explains the arrow of psychological time.

3.3.6 p-Adic physics as physics of cognition and intentionality

The sixth basic element adds a physical theory of cognition to this vision. TGD space-time decomposes into regions obeying real and p-adic topologies labelled by primes $p = 2, 3, 5, ...$ p-Adic regions obey the same field equations as the real regions but are characterized by p-adic non-determinism since the functions having vanishing p-adic derivative are pseudo constants which are piecewise constant functions. Pseudo constants depend on a finite number of positive pinary digits of arguments just like numerical predictions of any theory always involve decimal cutoff. This means that p-adic space-time regions are obtained by gluing together regions for which integration constants are genuine constants. The natural interpretation of the p-adic regions is as cognitive representations of real physics. The freedom of imagination is due to the p-adic non-determinism. p-Adic regions perform mimicry and make possible for the Universe to form cognitive representations about itself. p-Adic physics space-time sheets serve also as correlates for intentional action.

A more more precise formulation of this vision requires a generalization of the number concept obtained by fusing reals and p-adic number fields along common rationals (in the case of algebraic extensions among common algebraic numbers). This picture is discussed in [E1]. The application this notion at the level of the imbedding space implies that imbedding space has a book like structure with various variants of the imbedding space glued together along common rationals (algebraics). The implication is that genuinely p-adic numbers (non-rationals) are strictly infinite as real numbers so that most points of p-adic space-time sheets are at real infinity, outside the cosmos, and that the projection to the real imbedding space is discrete set of rationals (algebraics). Hence cognition and intentionality are almost completely outside the real cosmos and touch it at a discrete set of points only.

This view implies also that purely local p-adic physics codes for the p-adic fractality characterizing long range real physics and provides an explanation for p-adic length scale hypothesis stating that the primes $p \simeq 2^k$, k integer are especially interesting. It also explains the long range correlations and short term chaos characterizing intentional behavior and explains why the physical realizations of cognition are always discrete (say in the case of numerical computations). Furthermore, a concrete quantum model for how intentions are transformed to actions emerges.

The discrete real projections of p-adic space-time sheets serve also space-time correlate for a logical thought. It is very natural to assign to p-adic pinary digits a p-valued logic but as such this kind of logic does not have any reasonable identification. p-Adic length scale hypothesis suggest that the $p = 2^k - n$ pinary digits represent a Boolean logic B^k with k elementary statements (the points of the k-element set in the set theoretic realization) with n taboos which are constrained to be identically true.

3.4 TGD as a generalized number theory

Quantum T(opological)D(ynamics) as a classical spinor geometry for infinite-dimensional configuration space, p-adic numbers and quantum TGD, and TGD inspired theory of consciousness, have been for last ten years the basic three strongly interacting threads in the tapestry of quantum TGD. For few yeas ago the discussions with Tony Smith generated a fourth thread which deserves the name 'TGD as a generalized number theory'. It relies on the notion of number theoretic compactifiction stating that space-time surfaces can be regarded either as hyper-quaternionic, and thus maximally associative, 4-surfaces in M^8 identifiable as space of hyper-octonions or as surfaces in $M^4 \times CP_2$ [E2].

The discovery of the hierarchy of infinite primes and their correspondence with a hierarchy defined by a repeatedly second quantized arithmetic quantum field theory gave a further boost for the speculations about TGD as a generalized number theory. The work with Riemann hypothesis led to further ideas.

After the realization that infinite primes can be mapped to polynomials representable as surfaces geometrically, it was clear how TGD might be formulated as a generalized number theory with infinite primes forming the bridge between classical and quantum such that real numbers, p-adic numbers, and various generalizations of p-adics emerge dynamically from algebraic physics as various completions of the algebraic extensions of rational (hyper-)quaternions and (hyper-)octonions. Complete algebraic, topological and dimensional democracy would characterize the theory.

What is especially satisfying is that p-adic and real regions of the space-time surface could emerge automatically as solutions of the field equations. In the space-time regions where the solutions of field equations give rise to in-admissible complex values of the imbedding space coordinates, p-adic solution can exist for some values of the p-adic prime. The characteristic non-determinism of the p-adic differential equations suggests strongly that p-adic regions correspond to 'mind stuff', the regions of space-time where cognitive representations reside. This interpretation implies that p-adic physics is physics of cognition. Since Nature is probably extremely brilliant simulator of Nature, the natural idea is to study the p-adic physics of the cognitive representations to derive information about the real physics. This view encouraged by TGD inspired theory of consciousness clarifies difficult interpretational issues and provides a clear interpretation for the predictions of p-adic physics.

3.5 Dynamical quantized Planck constant and dark matter hierarchy

By quantum classical correspondence space-time sheets can be identified as quantum coherence regions. Hence the fact that they have all possible size scales more or less unavoidably implies that Planck constant must be quantized and have arbitrarily large values. If one accepts this then also the idea about dark matter as a macroscopic quantum phase characterized by an arbitrarily large value of Planck constant emerges naturally as does also the interpretation for the long ranged classical electro-weak and color fields predicted by TGD. Rather seldom the evolution of ideas follows simple linear logic, and this was the case also now. In any case, this vision represents the fifth, relatively new thread in the evolution of TGD and the ideas involved are still evolving.

3.5.1 Dark matter as large $\hbar$ phase

D. Da Rocha and Laurent Nottale [35] have proposed that Schrödinger equation with Planck constant $\hbar$ replaced with what might be called gravitational Planck constant $\hbar_{gr} = \frac{GmM}{v_0}$ ($\hbar = c = 1$). v_0 is a velocity parameter having the value $v_0 = 144.7 \pm .7$ km/s giving $v_0/c = 4.6 \times 10^{-4}$. This is rather near to the peak orbital velocity of stars in galactic halos. Also subharmonics and harmonics of v_0 seem to appear. The support for the hypothesis coming from empirical data is impressive.

Nottale and Da Rocha believe that their Schrödinger equation results from a fractal hydrodynamics. Many-sheeted space-time however suggests astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The gravitational (ordinary) Schrödinger equation would provide a solution of the black hole collapse (IR catastrophe) problem encountered at the classical level. The resolution of the problem inspired by TGD inspired theory of living matter is that it is the dark matter at larger space-time sheets which is quantum coherent in the required time scale [D6].

Already before learning about Nottale's paper I had proposed the possibility that Planck constant is quantized [E9] and the spectrum is given in terms of logarithms of Beraha numbers: the lowest Beraha number B_3 is completely exceptional in that it predicts infinite value of Planck constant. The inverse of the gravitational Planck constant could correspond a gravitational perturbation of this as $1/\hbar_{gr} = v_0/GMm$. The general philosophy would be that when the quantum system would become non-perturbative, a phase transition increasing the value of $\hbar$ occurs to preserve the perturbative character and at the transition $n = 4 \rightarrow 3$ only the small perturbative correction to $1/\hbar(3) = 0$ remains. This would apply to QCD and to atoms with $Z > 137$ as well.

TGD predicts correctly the value of the parameter v_0 assuming that cosmic strings and their decay remnants are responsible for the dark matter. The harmonics of v_0 can be understood as corresponding to perturbations replacing cosmic strings with their n-branched coverings so that tension becomes n^2-fold: much like the replacement of a closed orbit with an orbit closing only after n turns. $1/n$-sub-harmonic would result when a magnetic flux tube split into n disjoint magnetic flux tubes. Also a model for the formation of planetary system as a condensation of ordinary matter around quantum coherent dark matter emerges [D6].

3.5.2 Dark matter as a source of long ranged weak and color fields

Long ranged classical electro-weak and color gauge fields are unavoidable in TGD framework. The smallness of the parity breaking effects in hadronic, nuclear, and atomic length scales does not however seem to allow long ranged electro-weak gauge fields. The problem disappears if long range classical electro-weak gauge fields are identified as space-time correlates for massless gauge fields created by dark matter. Also scaled up variants of ordinary electro-weak particle spectra are possible. The identification explains chiral selection in living matter and unbroken $U(2)_{ew}$ invariance and free color in bio length scales become characteristics of living matter and of bio-chemistry and bio-nuclear physics. An attractive solution of the matter antimatter asymmetry is based on the identification of also antimatter as dark matter.

3.5.3 p-Adic and dark matter hierarchies and hierarchy of moments of consciousness

Dark matter hierarchy assigned to a spectrum of Planck constant having arbitrarily large values brings additional elements to the TGD inspired theory of consciousness.

a) Macroscopic quantum coherence can be understood since a particle with a given mass can in principle appear as arbitrarily large scaled up copies (Compton length scales as $\hbar$). The phase transition to this kind of phase implies that space-time sheets of particles overlap and this makes possible macroscopic quantum coherence.

b) The space-time sheets with large Planck constant can be in thermal equilibrium with ordinary ones without the loss of quantum coherence. For instance, the cyclotron energy scale associated with EEG turns out to be above thermal energy at room temperature for the level of dark matter hierarchy corresponding to magnetic flux quanta of the Earth's magnetic field with the size scale of Earth and a successful quantitative model for EEG results [M3].

Dark matter hierarchy leads to detailed quantitative view about quantum biology with several testable predictions [M3]. The applications to living matter suggests that the basic hierarchy corresponds to a hierarchy of Planck constants coming as $\hbar(k) = \lambda^k(p)\hbar_0$, $\lambda \simeq 2^{11}$ for $p = 2^{127-1}$, $k = 0, 1, 2, ...$ [M3]. Also integer valued sub-harmonics and integer valued sub-harmonics of λ might be possible. Each p-adic length scale corresponds to this kind of hierarchy and number theoretical arguments suggest a general formula for the allowed values of Planck constant λ depending logarithmically on p-adic prime [A8]. Also the value of $\hbar_0$ has spectrum characterized by Beraha numbers $B_n = 4cos^2(\pi/n)$, $n \geq 3$, varying by a factor in the range $n > 3$ [A8]. It must be however emphasized that the relation of this picture to the model of quantized gravitational Planck constant h_{gr} appearing in Nottale's model is not yet completely understood.

The general prediction is that Universe is a kind of inverted Mandelbrot fractal for which each bird's eye of view reveals new structures in long length and time scales representing scaled down copies of standard physics and their dark variants. These structures would correspond to higher levels in self hierarchy. This prediction is consistent with the belief that 75 per cent of matter in the universe is dark.

1. Living matter and dark matter

Living matter as ordinary matter quantum controlled by the dark matter hierarchy has turned out to be a particularly successful idea. The hypothesis has led to models for EEG predicting correctly the band structure and even individual resonance bands and also generalizing the notion of EEG [M3]. Also a generalization of the notion of genetic code emerges resolving the paradoxes related to the standard dogma [L2, M3]. A particularly fascinating implication is the possibility to identify great leaps in evolution as phase transitions in which new higher level of dark matter emerges [M3].

It seems safe to conclude that the dark matter hierarchy with levels labelled by the values of Planck constants explains the macroscopic and macro-temporal quantum coherence naturally.

That this explanation is consistent with the explanation based on spin glass degeneracy is suggested by following observations. First, the argument supporting spin glass degeneracy as an explanation of the macro-temporal quantum coherence does not involve the value of $\hbar$ at all. Secondly, the failure of the perturbation theory assumed to lead to the increase of Planck constant and formation of macroscopic quantum phases could be precisely due to the emergence of a large number of new degrees of freedom due to spin glass degeneracy. Thirdly, the phase transition increasing Planck constant has concrete topological interpretation in terms of many-sheeted space-time consistent with the spin glass degeneracy.

2. Dark matter hierarchy and the notion of self

The vision about dark matter hierarchy leads to a more refined view about self hierarchy and hierarchy of moments of consciousness [J6, M3]. The larger the value of Planck constant, the longer the subjectively experienced duration and the average geometric duration $T(k) \propto \lambda^k$ of the quantum jump.

Quantum jumps form also a hierarchy with respect to p-adic and dark hierarchies and the geometric durations of quantum jumps scale like $\hbar$. Dark matter hierarchy suggests also a slight modification of the notion of self. Each self involves a hierarchy of dark matter levels, and one is led to ask whether the highest level in this hierarchy corresponds to single quantum jump rather than a sequence of quantum jumps. The averaging of conscious experience over quantum jumps would occur only for sub-selves at lower levels of dark matter hierarchy and these mental images would be ordered, and single moment of consciousness would be experienced as a history of events. The quantum parallel dissipation at the lower levels would give rise to the experience of flow of time. For instance, hadron as a macro-temporal quantum system in the characteristic time scale of hadron is a dissipating system at quark and gluon level corresponding to shorter p-adic time scales. One can ask whether even entire life cycle could be regarded as a single quantum jump at the highest level so that consciousness would not be completely lost even during deep sleep. This would allow to understand why we seem to know directly that this biological body of mine existed yesterday.

The fact that we can remember phone numbers with 5 to 9 digits supports the view that self corresponds at the highest dark matter level to single moment of consciousness. Self would experience the average over the sequence of moments of consciousness associated with each sub-self but there would be no averaging over the separate mental images of this kind, be their parallel or serial. These mental images correspond to sub-selves having shorter wake-up periods than self and would be experienced as being time ordered. Hence the digits in the phone number are experienced as separate mental images and ordered with respect to experienced time.

3. The time span of long term memories as signature for the level of dark matter hierarchy

The simplest dimensional estimate gives for the average increment τ of geometric time in quantum jump $\tau \sim 10^4$ CP_2 times so that $2^{127} - 1 \sim 10^{38}$ quantum jumps are experienced during secondary p-adic time scale $T_2(k = 127) \simeq 0.1$ seconds which is the duration of physiological moment and predicted to be fundamental time scale of human consciousness [L1]. A more refined guess is that $\tau_p = \sqrt{p}\tau$ gives the dependence of the duration of quantum jump on p-adic prime p. By multi-p-fractality predicted by TGD and explaining p-adic length scale hypothesis, one expects that at least $p = 2$-adic level is also always present. For the higher levels of dark matter hierarchy τ_p is scaled up by $\hbar/\hbar_0$. One can understand evolutionary leaps as the emergence of higher levels at the level of individual organism making possible intentionality and memory in the time scale defined τ [L2].

Higher levels of dark matter hierarchy provide a neat quantitative view about self hierarchy and its evolution. For instance, EEG time scales corresponds to $k = 4$ level of hierarchy and a time scale of .1 seconds [J6], and EEG frequencies correspond at this level dark photon energies above the thermal threshold so that thermal noise is not a problem anymore. Various levels of dark matter hierarchy would naturally correspond to higher levels in the hierarchy of consciousness and the typical duration of life cycle would give an idea about the level in question.

The level would determine also the time span of long term memories as discussed in [M3]. $k = 7$ would correspond to a duration of moment of conscious of order human lifetime which suggests that $k = 7$ corresponds to the highest dark matter level relevant to our consciousness whereas higher levels would in general correspond to transpersonal consciousness. $k = 5$ would correspond to time scale of short term memories measured in minutes and $k = 6$ to a time scale of memories measured in days.

The emergence of these levels must have meant evolutionary leap since long term memory is also accompanied by ability to anticipate future in the same time scale. This picture would suggest that the basic difference between us and our cousins is not at the level of genome as it is usually understood but at the level of the hierarchy of magnetic bodies [L2, M3]. In fact, higher levels of dark matter hierarchy motivate the introduction of the notions of super-genome and hyper-genome. The genomes of entire organ can join to form super-genome expressing genes coherently. Hyper-genomes would result from the fusion of genomes of different organisms and collective levels of consciousness would express themselves via hyper-genome and make possible social rules and moral.

4 Bird's eye of view about the topics of the book

This book is devoted to a detailed representation of what might be called quantum TGD. Quantum TGD relies on two different views about physics: physics as an infinite-dimensional spinor geometry and physics as a generalized number theory. The most important guiding principle is quantum classical correspondence whose most profound implications follow almost trivially from the basic structure of the classical theory forming an exact part of quantum theory.

4.1 Quantum classical correspondence

Quantum classical correspondence has turned out to be the most important guiding principle concerning the interpretation of the theory.

4.1.1 The implications deriving from the topology of space-time surface and from the properties of induced gauge fields

Quantum classical correspondence and the properties of the simplest extremals of Kähler action have served as the basic guideline in the attempts to understand the new physics predicted by TGD. The most dramatic predictions follow without even considering field equations in detail by using quantum classical correspondence and form the backbone of TGD and TGD inspired theory of living matter in particular.

The notions of many-sheeted space-time, topological field quantization and the notion of field/magnetic body, follow from simple topological considerations. The observation that space-time sheets can have arbitrarily large sizes and their interpretation as quantum coherence regions forces to conclude that in TGD Universe macroscopic and macro-temporal quantum coherence are possible in arbitrarily long scales. It took relatively long time to realize that perhaps the only manner to understand this is a generalization of the quantum theory itself by allowing Planck constant to be dynamical and quantized. TGD leads indeed to a "prediction" for the spectrum of Planck constants and macroscopic quantum phases with large value of Planck constant allow an identification as a dark matter hierarchy.

Also long ranged classical color and electro-weak fields are an unavoidable prediction and it took a considerable time to make the obvious conclusion: the physics of TGD Universe is a fractal containing fractal copies of standard model physics at various space-time sheets labelled by the collection of p-adic primes assignable to elementary particles and by the level of dark matter hierarchy defined as $\hbar = \lambda^k \hbar_0$, $k_d = 0, 1, ...$. λ depends logarithmically on p-adic length scale $L(k)$ and satisfies $\lambda \simeq 2^{11}$ in atomic length scale $L(k = 137)$. Dark space-time sheets are identifiable as space-time sheets defining locally λ^k-fold covering of M^4 factor of the imbedding space so that darkness is a purely topological notion.

The new view about energy and time means that the sign of the inertial energy depends on the time orientation of the space-time sheet and that negative energy space-time sheets serve as correlates for communications to the geometric future. This alone leads to profoundly new views about metabolism, long term memory, and realization of intentional action.

4.1.2 Vacuum degeneracy of Kähler action as a correlate for quantum criticality and 4-dimensional spin glass degeneracy

The general properties of Kähler action, in particular its vacuum degeneracy and the failure of the classical determinism in the conventional sense, have also strong implications. Space-time surface as a generalization of Bohr orbit provides not only a representation of quantum states but also of sequences of quantum jumps and thus contents of consciousness. Vacuum degeneracy implies spin glass degeneracy in 4-D sense reflecting quantum criticality which is the fundamental characteristic of TGD Universe.

4.1.3 The simplest extremals of Kähler action as correlates for asymptotic self organization patterns

The detailed study of the simplest extremals of Kähler action interpreted as correlates for asymptotic self organization patterns provides additional insights [D1]. CP_2 type extremals representing elementary particles, cosmic strings, vacuum extremals, topological light rays ("massless extremal", ME), flux quanta of magnetic and electric fields represent the basic extremals. Pairs of wormhole throats identifiable as parton pairs define a completely new kind of particle carrying only color quantum numbers in ideal case and I have proposed their interpretation as quantum correlates for Boolean cognition. MEs and flux quanta of magnetic and electric fields are of special importance in living matter.

Topological light rays have interpretation as space-time correlates of "laser beams" of ordinary or dark photons or their electro-weak and gluonic counterparts. Neutral MEs carrying em and Z^0 fields are ideal for communication purposes and charged W MEs ideal for quantum control. Magnetic flux quanta containing dark matter are identified as intentional agents quantum controlling the behavior of the corresponding biological body parts utilizing negative energy W MEs. Bio-system in turn is populated by electrets identifiable as electric flux quanta.

4.2 Physics as infinite-dimensional geometry in the "world of classical worlds"

Physics as infinite-dimensional Kähler geometry of the "world of classical worlds" with classical spinor fields representing the quantum states of the universe and gamma matrix algebra geometrizing fermionic statistics is the first vision.

The mere existence of infinite-dimensional non-flat Kähler geometry has impressive implications. Configuration space must decompose to a union of infinite-dimensional symmetric spaces labelled by zero modes having interpretation as classical dynamical degrees of freedom assumed in quantum measurement theory. Infinite-dimensional symmetric space has maximal isometry group identifiable as a generalization of Kac Moody group obtained by replacing finite-dimensional group with the group of canonical transformations of $\delta M^4_+ \times CP_2$, where δM^4_+ is the boundary of 4-dimensional future light-cone. The infinite-dimensional Clifford algebra of configuration space gamma matrices in turn can be expressed as direct sum of von Neumann algebras known as hyper-finite factors of type II_1 having very close connections with conformal field theories, quantum and braid groups, and topological quantum field theories.

4.3 Physics as a generalized number theory

Second vision is physics as a generalized number theory. This vision forces to fuse real physics and various p-adic physics to a single coherent whole having rational physics as their intersection and poses extremely strong conditions on real physics.

A further aspect of this vision is the reduction of the classical dynamics of space-time sheets to number theory with space-time sheets identified as what I have christened hyper-quaternionic sub-manifolds of hyper-octonionic imbedding space. Field equations would state that space-time surfaces are Kähler calibrations with Kähler action density reducing to a closed 4-form at space-time surfaces. Hence TGD would define a generalized topological quantum field theory with conserved Noether charges (in particular rest energy) serving as generalized topological invariants having extremum in the set of topologically equivalent 3-surfaces.

Infinite primes, integers, and rationals define the third aspect of this vision. The construction of infinite primes is structurally similar to a repeated second quantization of an arithmetic quantum field theory and involves also bound states. Infinite rationals can be also represented as space-time surfaces somewhat like finite numbers can be represented as space-time points.

The seven online books about TGD [1, 2, 4, 5, 3, 6, 7] and eight online books about TGD inspired theory of consciousness and quantum biology [10, 8, 9, 13, 11, 12, 14, 15] are warmly recommended for the reader willing to get overall view about what is involved.

5 The contents of the book

5.1 Basic extremals of the Kähler action

In this chapter the classical field equations associated with the Kähler action are studied. The study of the extremals of the Kähler action has turned out to be extremely useful for the development of TGD. Towards the end of year 2003 quite dramatic progress occurred in the understanding of field equations and it seems that field equations might be in well-defined sense exactly solvable.

5.1.1 General considerations

The vanishing of Lorentz 4-force for the induced Kähler field means that the vacuum 4-currents are in a mechanical equilibrium. Lorentz 4-force vanishes for all known solutions of field equations which inspires the hypothesis that all extremals or at least the absolute minima of Kähler action satisfy the condition. The vanishing of the Lorentz 4-force in turn implies local conservation of the ordinary energy momentum tensor. The corresponding condition is implied by Einstein's equations in General Relativity. The hypothesis would mean that the solutions of field equations are what might be called generalized Beltrami fields. The condition implies that vacuum currents can be non-vanishing only provided the dimension D_{CP_2} of the CP_2 projection of the space-time surface is less than four so that in the regions with $D_{CP_2} = 4$, Maxwell's vacuum equations are satisfied.

The hypothesis that Kähler current is proportional to a product of an arbitrary function ψ of CP_2 coordinates and of the instanton current generalizes Beltrami condition and reduces to it when electric field vanishes. Kähler current has vanishing divergence for $D_{CP_2} < 4$, and Lorentz

4-force indeed vanishes. The remaining task would be the explicit construction of the imbeddings of these fields and the demonstration that field equations can be satisfied.

Under additional conditions magnetic field reduces to what is known as Beltrami field. Beltrami fields are known to be extremely complex but highly organized structures. The natural conjecture is that topologically quantized many-sheeted magnetic and Z^0 magnetic Beltrami fields and their generalizations serve as templates for the helical molecules populating living matter, and explain both chirality selection, the complex linking and knotting of DNA and protein molecules, and even the extremely complex and self-organized dynamics of biological systems at the molecular level.

Field equations can be reduced to algebraic conditions stating that energy momentum tensor and second fundamental form have no common components (this occurs also for minimal surfaces in string models) and only the conditions stating that Kähler current vanishes, is light-like, or proportional to instanton current, remain and define the remaining field equations. The conditions guaranteing topologization to instanton current can be solved explicitly. Solutions can be found also in the more general case when Kähler current is not proportional to instanton current. On basis of these findings there are strong reasons to believe that classical TGD is exactly solvable.

5.1.2 Does TGD define a a generalized topological quantum field theory?

A long standing assumption has been that the principle selecting the unique space-time going through given 3-surface (not reducing to single space-like component however) is absolute minimization of Kähler action. The number theoretical considerations suggest a more refined and more local principle, in which maximization or minimization of the value occurs for regions where the sign of action density is definite [E2] and that these two variational principles define dual pairs of 4-surfaces. This variational principle allows to interpret space-time surfaces generalized calibrations for which Kähler action density defines a four-form worm the physical extremals so that theory can be said to be topologized.

In fact already the generalized Bohr orbit property leads one to suspect that classical TGD defines a topological field theory generalized in such a manner that various conserved Noether charges can be regarded as topological invariants as extrema of these invariants for a given topology of 3-surface. There are of course zero modes characterizing the shape and size of the four-surface involved. Perhaps the maxima of Kähler function with respect to the zero modes define genuine topological invariants.

5.1.3 Generalized Bohr orbit property and second law of thermodynamics?

By quantum classical correspondence the non-deterministic space-time dynamics should mimic the dissipative dynamics of the quantum jump sequence. Beltrami fields appear in physical applications as asymptotic self organization patterns for which Lorentz force and dissipation vanish. This suggests that the preferred extrema defining the Bohr orbits, be they absolute minima of Kähler action or something more general, correspond to space-time sheets which asymptotically satisfy generalized Beltrami conditions so that one can indeed assign to the final (rather than initial!) 3-surface a unique 4-surface apart from effects related to non-determinism. Absolute minimization makes sense p-adically only if abstracted to purely algebraic generalized Beltrami conditions. The notion of Kähler calibration is purely algebraic local notion and certainly makes sense also p-adically. Also the equivalence of Bohr orbit property with the second law strongly suggests itself.

5.1.4 The dimension of CP_2 projection as classifier for the fundamental phases of matter

The dimension D_{CP_2} of CP_2 projection of the space-time sheet encountered already in p-adic mass calculations classifies the fundamental phases of matter. For $D_{CP_2} = 4$ empty space Maxwell equations hold true. This phase is chaotic and analogous to de-magnetized phase. $D_{CP_2} = 2$ phase is analogous to ferromagnetic phase: highly ordered and relatively simple. $D_{CP_2} = 3$ is the analog of spin glass and liquid crystal phases, extremely complex but highly organized by the properties of the generalized Beltrami fields. This phase is the boundary between chaos and order and corresponds to life emerging in the interaction of magnetic bodies with bio-matter. It is possible only in a finite temperature interval (note however the p-adic hierarchy of critical temperatures) and characterized by chirality just like life.

5.1.5 Specific extremals of Kähler action

The study of extremals of Kähler action represents more than decade old layer in the development of TGD.

1. The huge vacuum degeneracy is the most characteristic feature of Kähler action (any 4-surface having CP_2 projection which is Legendre sub-manifold is vacuum extremal, Legendre sub-manifolds of CP_2 are in general 2-dimensional). This vacuum degeneracy is behind the spin glass analogy and leads to the p-adic TGD. As found in the second part of the book, various particle like vacuum extremals also play an important role in the understanding of the quantum TGD.

2. The so called CP_2 type vacuum extremals have finite, negative action and are therefore an excellent candidate for real particles whereas vacuum extremals with vanishing Kähler action are candidates for the virtual particles. These extremals have one dimensional M^4 projection, which is light like curve but not necessarily geodesic and locally the metric of the extremal is that of CP_2: the quantization of this motion leads to Virasoro algebra. Space-times with topology $CP_2\#CP_2\#...CP_2$ are identified as the generalized Feynmann diagrams with lines thickened to 4-manifolds of "thickness" of the order of CP_2 radius. The quantization of the random motion with light velocity associated with the CP_2 type extremals in fact led to the discovery of Super Virasoro invariance, which through the construction of the configuration space geometry, becomes a basic symmetry of quantum TGD.

3. There are also various non-vacuum extremals.
i) String like objects, with string tension of same order of magnitude as possessed by the cosmic strings of GUTs, have a crucial role in TGD inspired model for the galaxy formation and in the TGD based cosmology.
ii) The so called massless extremals describe non-linear plane waves propagating with the velocity of light such that the polarization is fixed in given point of the space-time surface. The purely TGD:eish feature is the light like Kähler current: in the ordinary Maxwell theory vacuum gauge currents are not possible. This current serves as a source of coherent photons, which might play an important role in the quantum model of bio-system as a macroscopic quantum system.
iii) In the so called Maxwell's phase, ordinary Maxwell equations for the induced Kähler field are satisfied in an excellent approximation. A special case is provided by a radially symmetric extremal having an interpretation as the space-time exterior to a topologically condensed particle. The sign of the gravitational mass correlates with that of the Kähler charge and one can understand the generation of the matter antimatter asymmetry from the basic properties of this extremal. The possibility to understand the generation of the matter antimatter asymmetry directly from the basic equations of the theory gives strong support in favor of TGD in comparison to the ordinary EYM theories, where the generation of the matter antimatter asymmetry is still poorly understood.

5.2 Construction of Quantum Theory: Symmetries

This chapter provides a summary about the role of symmetries in the construction of quantum TGD. The discussions are based on the general vision that quantum states of the Universe correspond to the modes of classical spinor fields in the "world of the classical worlds" identified as the infinite-dimensional configuration space of light-like 3-surfaces of $H = M^4 \times CP_2$ (more or less-equivalently, the corresponding 4-surfaces defining generalized Bohr orbits). The following topics are discussed on basis of this vision.

1. Geometric ideas

TGD relies heavily on geometric ideas, which have gradually generalized during the years. Symmetries play a key role as one might expect on basis of general definition of geometry as a structure characterized by a given symmetry.

1.1 Physics as infinite-dimensional Kähler geometry

1. The basic idea is that it is possible to reduce quantum theory to configuration space geometry and spinor structure. The geometrization of loop spaces inspires the idea that the mere existence of Riemann connection fixes configuration space Kähler geometry uniquely. Accordingly, configuration space can be regarded as a union of infinite-dimensional symmetric spaces labelled by zero modes labelling classical non-quantum fluctuating degrees of freedom.

   The huge symmetries of the configuration space geometry deriving from the light-likeness of 3-surfaces and from the special conformal properties of the boundary of 4-D light-cone would guarantee the maximal isometry group necessary for the symmetric space property. Quantum criticality is the fundamental hypothesis allowing to fix the Kähler function and thus dynamics of TGD uniquely. Quantum criticality leads to surprisingly strong predictions about the evolution of coupling constants.

2. Configuration space spinors correspond to Fock states and anti-commutation relations for fermionic oscillator operators correspond to anti-commutation relations for the gamma matrices of the configuration space. Configuration space gamma matrices contracted with Killing vector fields give rise to a super-algebra which together with Hamiltonians of the configuration space forms what I have used to called super-canonical algebra.

   Super-canonical degrees of freedom represent completely new degrees of freedom and have no electroweak couplings. In the case of hadrons super-canonical quanta correspond to what has been identified as non-perturbative sector of QCD: they define TGD correlate for the degrees of freedom assignable to hadronic strings. They are responsible for the most of the mass of hadron and resolve spin puzzle of proton.

   Besides super-canonical symmetries there are Super-Kac Moody symmetries assignable to light-like 3-surfaces and together these algebras extend the conformal symmetries of string models to dynamical conformal symmetries instead of mere gauge symmetries. The construction of the representations of these symmetries is one of the main challenges of quantum TGD. Modular invariance is one aspect of conformal symmetries and plays a key role in the understanding of elementary particle vacuum functionals and the description of family replication phenomenon in terms of the topology of partonic 2-surfaces.

3. Configuration space spinors define a von Neumann algebra known as hyper-finite factor of type II_1 (HFFs). This realization has led also to a profound generalization of quantum TGD through a generalization of the notion of imbedding space to characterize quantum criticality. The resulting space has a book like structure with various almost-copies of imbedding space representing the pages of the book meeting at quantum critical sub-manifolds. The outcome

of this approach is that the exponents of Kähler function and Chern-Simons action are not fundamental objects but reduce to the Dirac determinant associated with the modified Dirac operator assigned to the light-like 3-surfaces.

1.2 p-adic physics and p-adic variants of basic symmetries

p-Adic mass calculations relying on p-adic length scale hypothesis led to an understanding of elementary particle masses using only super-conformal symmetries and p-adic thermodynamics. The need to fuse real physics and various p-adic physics to single coherent whole led to a generalization of the notion of number obtained by gluing together reals and p-adics together along common rationals and algebraics. The interpretation of p-adic space-time sheets is as correlates for cognition and intentionality. p-Adic and real space-time sheets intersect along common rationals and algebraics and the subset of these points defines what I call number theoretic braid in terms of which both configuration space geometry and S-matrix elements should be expressible. Thus one would obtain number theoretical discretization which involves no adhoc elements and is inherent to the physics of TGD.

1.3. Hierarchy of Planck constants and dark matter hierarchy

The work with HFFs combined with experimental input led to the notion of hierarchy of Planck constants interpreted in terms of dark matter. The hierarchy is realized via a generalization of the notion of imbedding space obtained by gluing infinite number of its variants along common lower-dimensional quantum critical sub-manifolds. These variants of imbedding space are characterized by discrete subgroups of SU(2) acting in M^4 and CP_2 degrees of freedom as either symmetry groups or homotopy groups of covering. Among other things this picture implies a general model of fractional quantum Hall effect.

This picture also leads to the identification of number theoretical braids as points of partonic 2-surface which correspond to the minima of generalized eigenvalue of Dirac operator, a scalar field to which Higgs vacuum expectation is proportional to. Higgs vacuum expectation has thus a purely geometric interpretation. The outcome is an explicit formula for the Dirac determinant consistent with the vacuum degeneracy of Kähler action and its finiteness and algebraic number property required by p-adicization by number theoretic universality.

What is especially remarkable is that the construction gives also the 4-D space-time sheets associated with the light-like orbits of partonic 2-surfaces: it remains to be shown whether they correspond to preferred extremals of Kähler action. One can conclude that the hierarchy of Planck constants is now an essential part of construction of quantum TGD and of mathematical realization of the notion of quantum criticality rather than a possible generalization of TGD.

1.4. Number theoretical symmetries

TGD as a generalized number theory vision leads to the idea that also number theoretical symmetries are important for physics.

1. There are good reasons to believe that the strands of number theoretical braids can be assigned with the roots of a polynomial with suggests the interpretation corresponding Galois groups as purely number theoretical symmetries of quantum TGD. Galois groups are subgroups of the permutation group S_∞ of infinitely manner objects acting as the Galois group of algebraic numbers. The group algebra of S_∞ is HFF which can be mapped to the HFF defined by configuration space spinors. This picture suggest a number theoretical gauge invariance stating that S_∞ acts as a gauge group of the theory and that global gauge transformations in its completion correspond to the elements of finite Galois groups represented as diagonal groups of $G \times G \times ....$ of the completion of S_∞.

2. HFFs inspire also an idea about how entire TGD emerges from classical number fields, actually their complexifications. In particular, SU(3) acts as subgroup of octonion automorphisms leaving invariant preferred imaginary unit and $M^4 \times CP_2$ can be interpreted as a structure related to hyper-octonions which is a subspace of complexified octonions for which metric has naturally Minkowski signature. This would mean that TGD could be seen also as a generalized number theory. This conjecture predicts the existence of two dual formulations of TGD based on the identification space-times as 4-surfaces in hyper-octonionic space M^8 *resp.* $M^4 \times CP_2$.

2. The construction of S-matrix

The construction of S-matrix involves several ideas that have emerged during last years and involve symmetries in an essential manner.

2.1 Zero energy ontology

Zero energy ontology motivated originally by TGD inspired cosmology means that physical states have vanishing conserved net quantum numbers and are decomposable to positive and negative energy parts separated by a temporal distance characterizing the system as a space-time sheet of finite size in time direction. The particle physics interpretation is as initial and final states of a particle reaction. Obviously a profound modification of existing views about realization of symmetries is in question.

S-matrix and density matrix are unified to the notion of M-matrix defining time-like entanglement and expressible as a product of square root of density matrix and of unitary S-matrix. Thermodynamics becomes therefore a part of quantum theory. One must distinguish M-matrix from U-matrix defined between zero energy states and analogous to S-matrix and characterizing the unitary process associated with quantum jump. U-matrix is most naturally related to the description of intentional action since in a well-defined sense it has elements between physical systems corresponding to different number fields.

2.2 Quantum TGD as almost topological QFT

Light-likeness of the basic fundamental objects implies that TGD is almost topological QFT so that the formulation in terms of category theoretical notions is expected to work. M-matrices form in a natural manner a functor from the category of cobordisms to the category of pairs of Hilbert spaces and this gives additional strong constraints on the theory. Super-conformal symmetries implied by the light-likeness pose very strong constraints on both state construction and on M-matrix and U-matrix. The notions of n-category and n-groupoid which represents a generalization of the notion of group could be very relevant to this view about M-matrix.

2.3. Quantum measurement theory with finite measurement resolution

The notion of measurement resolution represented in terms of inclusions $\mathcal{N} \subset \mathcal{M}$ of HFFs is an essential element of the picture. Measurement resolution corresponds to the action of the included sub-algebra creating zero energy states in time scales shorter than the cutoff scale. This means that complex rays of state space are effectively replaced with $\mathcal{N}$ rays. The condition that the action of $\mathcal{N}$ commutes with the M-matrix is a powerful symmetry and implies that the time-like entanglement characterized by M-matrix corresponds to Connes tensor product. Together with super-conformal symmetries this symmetry should fix possible M-matrices to a very high degree.

The notion of number theoretical braid realizes the notion of finite measurement resolution at space-time level and gives a direct connection to topological QFTs describing braids. The connection with quantum groups is highly suggestive since already the inclusions of HFFs involve these groups. Effective non-commutative geometry for the quantum critical sub-manifolds $M^2 \subset M^4$

and $S^2 \subset CP_2$ might provide an alternative notion for the reduction of stringy anti-commutation relations for induced spinor fields to anti-commutations at the points of braids.

5.3 Construction of Quantum Theory: S-matrix

The construction of S-matrix has been key challenge of quantum TGD from the very beginning when it had become clear that path integral approach and canonical quantization make no sense in TGD framework. My intuitive feeling that the problems are not merely technical has turned out to be correct. In this chapter the overall view about the construction of S-matrix is discussed. It is perhaps wise to summarize briefly the vision about S-matrix.

1. S-matrix defines entanglement between positive and negative energy parts of zero energy states. This kind of S-matrix need not be unitary unlike the U-matrix associated with unitary process forming part of quantum jump. There are several good arguments suggesting that that S-matrix cannot unitary but can be regarded as thermal S-matrix so that thermodynamics would become an essential part of quantum theory. In TGD framework path integral formalism is given up although functional integral over the 3-surfaces is present.

2. Almost topological QFT property of quantum allows to identify S-matrix as a functor from the category of generalized Feynman cobordisms to the category of operators mapping the Hilbert space of positive energy states to that for negative energy states: these Hilbert spaces are assignable to partonic 2-surfaces. Feynman cobordism is the generalized Feynman diagram having light-like 3-surfaces as lines glued together along their ends defining vertices as 2-surfaces. This picture differs dramatically from that of string models. There is functional integral over the small deformations of Feynman cobordisms corresponding to maxima of Kähler function.

3. Imbedding space degrees of freedom seem to imply the presence of factor of type I besides HFF of type II_1 for which unitary S-matrix can define time-like entanglement coefficients. Only thermal S-matrix defines a normalizable zero energy state so that thermodynamics becomes part of quantum theory. One can assign to S-matrix a complex parameter whose real part has interpretation as interaction time and imaginary part as the inverse temperature. S-matrices and thus also quantum states in zero energy ontology possess a semigroup like structure and in the product time and inverse temperature are additive. This suggests that the cosmological evolution of temperature as $T \propto 1/t$ could be understood at the level of fundamental quantum theory.

4. S-matrix should be constructible as a generalization of braiding S-matrix in such a manner that the number theoretic braids assignable to light-like partonic 3-surfaces glued along their ends at 2-dimensional partonic 2-surfaces representing reaction vertices replicate in the vertex.

5. The construction of braiding S-matrices assignable to the incoming and outgoing partonic 2-surfaces is not a problem. The problem is to express mathematically what happens in the vertex. Here the observation that the tensor product of HFFs of type II is HFF of type II is the key observation. Many-parton vertex can be identified as a unitary isomorphism between the tensor product of incoming *resp.* outgoing HFFs. A reduction to HFF of type II_1 occurs because only a finite-dimensional projection of S-matrix in bosonic degrees of freedom defines a normalizable state. In the case of factor of type II_∞ only thermal S-matrix is possible without finite-dimensional projection and thermodynamics would thus emerge as an essential part of quantum theory.

6. HFFs of type III could also appear at the level of field operators used to create states but at the level of quantum states everything reduces to HFFs of type II_1 and their tensor products giving these factors back. If braiding automorphisms reduce to the purely intrinsic unitary automorphisms of HFFs of type III then for certain values of the time parameter of automorphism having interpretation as a scaling parameter these automorphisms are trivial. These time scales could correspond to p-adic time scales so that p-adic length scale hypothesis would emerge at the fundamental level. In this kind of situation the braiding S-matrices associated with the incoming and outgoing partons could be trivial so that everything would reduce to this unitary isomorphism: a counterpart for the elimination of external legs from Feynman diagram in QFT.

7. One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This turns out to be the case only in non-perturbative phase. The reason is that the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be regarded as negligible and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement.

8. In this picture light-like 3-surface would take the dual role as a correlate for both state and time evolution of state and this dual role allows to understand why the restriction of time like entanglement to that described by S-matrix must be made. For fixed values of moduli each reaction would correspond to a minimal braid diagram involving exchanges of partons being in one-one correspondence with a maximum of Kähler function. By quantum criticality and the requirement of ideal quantum-classical correspondence only one such diagram would contribute for given values of moduli. Coupling constant evolution would not be however lost: it would be realized as p-adic coupling constant at the level of free states via the $log(p)$ scaling of eigen modes of the modified Dirac operator.

9. A completely unexpected prediction deserving a special emphasis is that number theoretic braids replicate in vertices. This is of course the braid counterpart for the introduction of annihilation and creation of particles in the transition from free QFT to an interacting one. This means classical replication of the number theoretic information carried by them. This allows to interpret one of the TGD inspired models of genetic code in terms of number theoretic braids representing at deeper level the information carried by DNA. This picture provides also further support for the proposal that DNA acts as topological quantum computer utilizing braids associated with partonic light-like 3-surfaces (which can have arbitrary size). In the reverse direction one must conclude that even elementary particles could be information processing and communicating entities in TGD Universe.

5.4 Category Theory and Quantum TGD

Possible applications of category theory to quantum TGD are discussed. The so called 2-plectic structure generalizing the ordinary symplectic structure by replacing symplectic 2-form with 3-form and Hamiltonians with Hamiltonian 1-forms has a natural place in TGD since the dynamics of the light-like 3-surfaces is characterized by Chern-Simons type action. The notion of planar operad was developed for the classification of hyper-finite factors of type II_1 and its mild generalization allows to understand the combinatorics of the generalized Feynman diagrams obtained by gluing 3-D light-like surfaces representing the lines of Feynman diagrams along their 2-D ends representing the vertices.

The fusion rules for the symplectic variant of conformal field theory, whose existence is strongly suggested by quantum TGD, allow rather precise description using the basic notions of category theory and one can identify a series of finite-dimensional nilpotent algebras as discretized versions of field algebras defined by the fusion rules. These primitive fusion algebras can be used to construct more complex algebras by replacing any algebra element by a primitive fusion algebra. Trees with arbitrary numbers of branches in any node characterize the resulting collection of fusion algebras forming an operad. One can say that an exact solution of symplectic scalar field theory is obtained.

Conformal fields and symplectic scalar field can be combined to form symplecto-formal fields. The combination of symplectic operad and Feynman graph operad leads to a construction of Feynman diagrams in terms of n-point functions of conformal field theory. M-matrix elements with a finite measurement resolution are expressed in terms of a hierarchy of symplecto-conformal n-point functions such that the improvement of measurement resolution corresponds to an algebra homomorphism mapping conformal fields in given resolution to composite conformal fields in improved resolution. This expresses the idea that composites behave as independent conformal fields. Also other applications are briefly discussed.

5.5 Hyper-Finite Factors and Construction of S-Matrix

During years I have spent a lot of time and effort to attempts to imagine various options for the construction of S-matrix. Contrary to my original belief, the real problem has not been the lack of my analytic skills but the failure of ordinary QFT based thinking in TGD framework.

Super-conformal symmetries generalized from string model context to TGD framework are symmetries of S-matrix. This is very powerful constraint to S-matrix but useless unless one has precisely defined ontology translated to a rigorous mathematical framework. The zero energy ontology of TGD is now rather well understood but differs dramatically from that of standard quantum field theories. Second deep difference is that path integral formalism is given up and the goal is to construct S-matrix as a generalization of braiding S-matrices with reaction vertices replaced with the replication of number theoretic braids associated with partonic 2-surfaces taking the role of vertices. Also number theoretic universality requiring fusion of real physics and various p-adic physics to single coherent whole is a completely new element.

The most recent vision about S-matrix combines ideas scattered in various chapters of various books and often drowned with details. A very brief summary would be as follows.

1. In TGD framework functional integral formalism is given up. S-matrix should be constructible as a generalization of braiding S-matrix in such a manner that the number theoretic braids assignable to light-like partonic 3-surfaces glued along their ends at 2-dimensional partonic 2-surfaces representing reaction vertices replicate in the vertex. This means a replacement of the free dynamics of point particles of quantum field theories with braiding dynamics associated with partonic 2-surfaces carrying braids and the replacement of particle creation with the creation of partons and replication of braids.

2. The construction of braiding S-matrices assignable to the incoming and outgoing partonic 2-surfaces is not a problem. The problem is to express mathematically what happens in the vertex. Here the observation that the tensor product of hyper-finite factors (HFFs) of type II is HFF of type II is the key observation. Many-parton vertex can be identified as a unitary isomorphism between the tensor product of incoming *resp.* outgoing HFFs. A reduction to HFF of type II_1 occurs because only a finite-dimensional projection of S-matrix in bosonic degrees of freedom defines a normalizable state. Most importantly, unitarity and non-triviality of S-matrix follows trivially.

3. HFFs of type III could also appear at the level of field operators used to create states but that at the level of quantum states everything reduces to HFFs of type II_1 and their tensor

products giving these factors back. If braiding automorphisms reduce to the purely intrinsic unitary automorphisms of HFFs of type III then for certain values of the time parameter of automorphism having interpretation as a scaling parameter these automorphisms are trivial. These time scales could correspond to p-adic time scales so that p-adic length scale hypothesis would emerge at the fundamental level. In this kind of situation the braiding S-matrices associated with the incoming and outgoing partons could be trivial so that everything would reduce to this unitary isomorphism: a counterpart for the elimination of external legs from Feynman diagram in QFT. p-Adic thermodynamics and particle massivation could be also obtained when the time parameter of the automorphism is allowed to be complex as a generalization of thermal QFT.

4. One might hope that all complications related to what happens for *space-like* 3-surfaces could be eliminated by quantum classical correspondence stating that space-time view about particle reaction is only a space-time correlate for what happens in quantum fluctuating degrees of freedom associated with partonic 2-surfaces. This turns out to be the case only in non-perturbative phase. The reason is that the arguments of n-point function appear as continuous moduli of Kähler function. In non-perturbative phases the dependence of the maximum of Kähler function on the arguments of n-point function cannot be regarded as negligible and Kähler function becomes the key to the understanding of these effects including formation of bound states and color confinement.

5. In this picture light-like 3-surface would take the dual role as a correlate for both state and time evolution of state and this dual role allows to understand why the restriction of time like entanglement to that described by S-matrix must be made. For fixed values of moduli each reaction would correspond to a minimal braid diagram involving exchanges of partons being in one-one correspondence with a maximum of Kähler function. By quantum criticality and the requirement of ideal quantum-classical correspondence only one such diagram would contribute for given values of moduli. Coupling constant evolution would not be however lost: it would be realized as p-adic coupling constant at the level of free states via the $log(p)$ scaling of eigen modes of the modified Dirac operator.

6. A completely unexpected prediction deserving a special emphasis is that number theoretic braids replicate in vertices. This means classical replication of the number theoretic information carried by them. This allows to interpret one of the TGD inspired models of genetic code in terms of number theoretic braids representing at deeper level the information carried by DNA. This picture provides also further support for the proposal that DNA acts as topological quantum computer utilizing braids associated with partonic light-like 3-surfaces (which can have arbitrary size). In the reverse direction one must conclude that even elementary particles could be information processing and communicating entities in TGD Universe.

To sum up, my personal feeling is that the constraints identified hitherto might lead to a more or less unique final result and I can only hope that some young analytically blessed brain would bother to transform this picture to concrete calculational recipes.

5.6 Earlier attempts to construct S-matrix

The dream of finding master formula for S-matrix of TGD is 27 year old as I write this. The realization that configuration space spinors correspond to von Neumann algebras known as hyper-finite factors of type II_1 was the decisive discovery and stimulated a rapid progress in the understanding of the mathematical structure of TGD and also the long waited master formula for S-matrix finally saw the daylight.

This master formula made however a lot of previous work obsolete. These earlier attempts might seem rather childish from the recent point of view which could of course look equally childish from the perspective I perhaps have around 2010. The germs of recent vision are however in these older attempts although strongly suppressed by cognitive noise. These older visions contain also a lot of real stuff which complements the recent vision. In particular, the section "Approximate construction of S-matrix" is to a surprisingly high degree consistent with master formula". For these reasons I thought that it would not be wise to throw away this chapter. I have not added the recent overall view about construction of S-matrix which is contained in the chapter "Construction of Quantum Theory" and is warmly recommended to the critical reader as a background.

The gigantic symmetries of quantum TGD are bound to lead to a highly unique S-matrix (actually a hierarchy of S-matrices) but the practical construction of U-matrix remains still a formidable challenge. Despite this one can write Feynman rules for the S-matrix in the approximation that the consideration is restricted to elementary particles modelled as CP_2 type extremals. Two developments have changed the situation dramatically.

1. The first development was inspired by the idea that generalized Feynman diagrams might allow a generalization of duality symmetry of string models meaning that the diagrams are always equivalent with tree diagrams. The diagrams could be seen as generalization of braid diagrams and this notion can be formulated axiomatically in terms of Hopf algebras.

2. The second development was inspired by the better understanding of the role of the classical non-determinism of Kähler action and led to the discovery of 7–3 duality and effective 2-dimensionality meaning that all relevant physics can be coded into 2-dimensional intersections of 3-D and 7-D light like causal determinants. This gives concrete realization for the equivalence of generalized diagrams with tree diagrams.

1. Basic philosophy behind the construction of S-matrix

In TGD framework quantum transitions correspond to a quantum jump between two different quantum histories (configuration space spinor field) rather than to a non-deterministic behavior of a single quantum history. Therefore S-matrix relates to each other two quantum histories rather than the initial and final states of a single quantum history.

To understand the philosophy behind the construction of S-matrix it is useful to notice that in TGD framework there is actually a 'holy trinity' of time developments instead of single time development encountered in ordinary quantum field theories.

1. The classical time development determined by the absolute minimization of Kähler action.

2. The unitary "time development" defined by U associated with each quantum jump and defining U-matrix. One cannot however assign to the U-matrix an interpretation as a unitary time-translation operator and this means that one must leave open the identification of U-matrix with S-matrix.

3. The time development of subjective experiences by quantum jumps identified as moments of consciousness. The value of psychological time associated with a given quantum jump is determined by the contents of consciousness of the observer. The understanding of psychological time and its arrow and of the dynamics of subjective time development requires the construction of theory of consciousness. A crucial role is played by the classical nondeterminism of Kähler action implying that the nondeterminism of quantum jump and hence also the contents of conscious experience can be concentrated into a finite volume of the imbedding space.

Quantum classical correspondence states that not only quantum states but also quantum jump sequences and even the complex anatomy of quantum jump must have representation at space-time level. This has far reaching implications.

1. Configuration space S-matrix or U-matrix is induced from space-time S-matrix acting in fermionic degrees of freedom (configuration space "spin degrees of freedom").
2. The fact that there is an infinite number of anatomies of quantum jump connecting given quantum states means, predicts that there is a infinite number of space-time surfaces giving rise to the same space-time S-matrix. This is nothing but the equivalence of generalized Feynman diagrams to tree diagrams.
3. Single quantum jump and thus a particular space-time S-matrix must correspond to a finite space-time region, perhaps single space-time sheet with the maximal deterministic regions of the space-time sheet correlating with with the anatomy of quantum jump.

2. 7–3 duality as a key to the construction of S-matrix

The notion of 7–3 duality emerged from the interaction between TGD and M-theory. The attempts to construct quantum TGD have gradually led to the conclusion that the geometry of the configuration space ("world of classical worlds") involves both 7-D and 3-D light like surfaces as causal determinants. 7-D light like surfaces X^7 are unions of future and past light cone boundaries and play a role somewhat resembling that of branes. 3-D light like surfaces X^3_l can correspond to boundaries of space-time sheets, regions separating two maximally deterministic space-time regions, and elementary particle horizons at which the signature of the induced metric changes.

7–3 duality states that it is possible to formulate the theory using either the data at 3-D space-like 3-surfaces resulting as intersections of the space-time surface with 7-D CDs or the data at 3-D light like CDs. This results if the data needed is actually contained by 2-D intersections $X^2 = X^3_l \cap X^7$. This effective 2-dimensionality has far-reaching implications. It simplifies dramatically the basic formulas related to the configuration space geometry and spinor structure, it leads to the explicit identification of the generalized Feynman diagrams at space-time level as light like 3-D CDs. The basic philosophy is that quantum-classical correspondence stating that space-time sheets provide a description for the physics associated with the configuration space spin degrees of freedom (fermionic degrees of freedom).

The generalized Feynman diagrammatics is simple. The fermions do not carry four-momenta but are on mass shell particles characterized by the eigenvalues of the modified Dirac operator D. There is no propagator associated with 3-D CDs: only a unitary transformation U_λ representing braiding in spin and electroweak spin degrees of freedom can be present. Vertices are the inner products at X^2 for the positive energy states and negative energy states entering to the vertex, finite, and in principle computable. The equivalence of generalized Feynman diagrams with tree diagrams is expected on basis of the effective 2-dimensionality, and indeed follows from on mass shell property directly. Unitarity follows trivially. No loop summations are thus involved.

The counterparts of loop sums are absent in TGD framework and p-adic number fields and their extensions defining an infinite hierarchy of fixed point values of Kähler coupling strength and thus of gauge coupling constants. The question is whether this discrete coupling constant evolution can mimic a QFT type coupling constant evolution (or vice versa). Is it possible to have renormalization without renormalization? The construction of quantum state using generalization of coset construction for super-canonical and super Kac-Moody algebra allows to answer this question. The counterparts of bare states are non-orthogonal and have a natural multi-grading. Gram-Schmidt orthogonalization procedure makes the bare states dressed and brings in TGD counterpart of loop corrections to the S-matrix. The counterparts of renormalization group equations result by formally regarding p-adic prime p as a continuous variable. Quantum field theory approximation results when the inner products defining simplest particle decays are described as coupling constants.

3. Quantum criticality and Hopf algebra approach to S-matrix

Quantum criticality leads to a generalization of duality symmetry of string models stating that the generalized Feynman diagrams with loops are equivalent with diagrams having no loops. This means that each S-matrix element correspond to a unique tree diagram. The conditions for this equivalence can be formulated as algebraic conditions characterizing a Hopf algebra like structure, and, using the language of ordinary Feynman diagrams, correspond to the vanishing of the loop corrections in the configuration space integral crucial for the p-adicization. This symmetry is expected to be of crucial importance for practical evaluation of S-matrix elements as should be also the reduction of the matrix elements of generators of the enveloping algebra of super-canonical algebra to n-point functions of a conformal field theory in the complex plane of super-canonical conformal weights.

4. S-matrix as Glebsch-Gordan coefficients

U-matrix relates 'free' and 'interacting' representations of the super-canonical and Super Kac-Moody algebras acting as symmetries of quantum TGD. The construction is based on the association of 3-surfaces Y_i^3 and corresponding absolute minima $X^4(Y_i^3)$ to incoming states as well as the interacting four-surface $X^4(\cup_i Y_i^3)$ describing the interactions classically. The generators for various super-algebras associated with $X^4(\cup_i Y_i^3)$ are modified by interactions so that the generator basis is not just a union of the generator basis associated with $X^4(Y_i^3)$. U-matrix relates the tensor product for the representations associated with the incoming 'free' space-time surfaces $X^3(Y_i^3)$ and the interaction representation associated with $X^4(\cup Y_i^3)$: generalized Glebch-Gordan coefficients are clearly in question and unitarity is obvious.

5. Perturbation theoretic approach to U-matrix

This formal approach starts from the identification of U-matrix elements as Glebsch-Gordan coefficients relating free and interacting states and tries to construct U-matrix perturbatively by reducing it to stringy perturbation theory. The starting point is that U-matrix must follow from Super Virasoro invariance alone and that the condition $L_0(tot)\Psi = 0$ (plus the corresponding conditions for other super-Virasoro generators) must determine U-matrix. Here $L_0(tot)$ corresponds to the Virasoro generators associated with the interacting space-time surface $X^4(\cup_i Y_i^3)$ whereas $L_0(free, i)$ correspond to the free generators associated with $X^3(Y_i^3)$. It is however not at all obvious whether the generators $L_0(tot)$ are perturbatively related to the the generators $L_0(free, i)$ and whether U-matrix allows perturbative expansion.

6. Number theoretic approach to U-matrix

The task of assigning to the surfaces Y_i^3 the free space-time surfaces $X^4(Y_i^3)$ and interacting space-time surface $X^4(\cup_i Y_i^3)$ is the basic stumbling block for the construction of U-matrix. The super-algebra generators creating the excitations of the incoming ground states are super-algebra generators associated with $\cup X^4(Y_i^3)$ whereas the outgoing states are created by the super-algebra generators associated with $X^4(\cup_i Y_i^3)$. The surfaces $X^4(Y_i^3)$ correspond to the space-time surfaces associated with infinite primes P_i representing ground states of super-conformal representations whereas $X^4(\cup_i Y_i^3)$ corresponds to the space-time surface associated with the infinite integer $N = \prod_i P_i^{k_i}$. This means that the worst part of the problem is solved. The remaining challenge is to relate the super-algebra basis to each other.

7. Construction of the S-matrix at high energy limit

It is possible to write Feynman rules for the S-matrix in the approximation that only CP_2 type extremals appear as virtual and real particles. All CP_2 type extremals are locally isometric with

CP_2 itself and only the random lightlike curve is dynamical. The classical dynamics is actually isomorphic with stringy dynamics since classical Virasoro conditions are satisfied. Fermions belong to the representations of Super-Kac-Moody algebra of $M^4 \times SO(3,1) \times SU(3) \times U(2)_{ew}$. The classical nondeterminism of the dynamics implies that Feynman graph expansion is topologized. This saves from the troubles caused by fermionic divergences since the exponent of the momentum generator effecting translation along the line of the Feynman graph corresponds to that associated with the modified Dirac action and thus to a free quantum theory for fermions.

Vertex operators $V(a,b,c)$ are generalizations of the vertex operators of string theory: instead of strings 3-surface inside CP_2 type extremal fuse together. Propagator factors are products of the exponent of the Kähler action for CP_2 type extremal proportional to the volume of the CP_2 type extremal; the 'stringy' $1/(L_0 + i\epsilon)$ factor, which comes from the vertices; and a unitary translation operator (counterpart of S-matrix as time translation operator) along the geodesic representing average cm motion.

The theory has some features which are characteristic for quantum TGD.

1. One can assume that each quantum jump involves localization in zitterbewegung degrees of freedom. The resulting S-matrix is independent of the choice of the representative for the zitterbewegung orbit as long as the cm motion connects the lines of the vertices. The predictions depend however on an arbitrary function of U of CP_2 coordinates giving rise to a decomposition of CP_2 to 'time slices'. The dependence of the propagator is only through the volume of CP_2 type extremal determined by U whereas coupling constants have more complicated, but presumably very mild dependence on U. The dependence on the function U means that one must average the scattering rates over the allowed spectrum of functions U. This dependence of the fundamental coupling constants on U is in accordance with spin glass analogy and means that fundamental coupling constants are not strictly speaking constants.

2. The volume of the internal line, which is a fraction of CP_2 volume determines the value of the exponent of Kähler action and provides thus a suppression factor serving as an infrared cutoff. A constraint to the allowed functions U results from the topological condensation of CP_2 in particle like space-time sheet (for instance, massless extremal), which implies that CP_2 type extremals cannot extend outside the region with size of order p-adic length scale L_p. The only plausible interpretation seems to be that the information about the infrared cutoff length scale is coded into the structure of particle: particle in the box is quite not the same as free particle. This suggests new view about color confinement: quarks and gluons correspond to CP_2 type extremals which cannot exist too long time as free particles and therefore cannot leave hadron.

5.7 Is it possible to understand coupling constant evolution at space-time level?

It is not yet possible to deduce the length scale evolution of gauge coupling constants from Quantum TGD proper although one can understand precisely the origin of the p-adic coupling constant evolution in the partonic formulation of quantum TGD as a almost topological super-conformal field theory. Quantum classical correspondence however encourages the hope that it might be possible to achieve some understanding of the coupling constant evolution by using the classical theory.

This turns out to be the case and the earlier speculative picture about gauge coupling constants associated with a given space-time sheet as RG invariants finds support. It remains an open question whether gravitational coupling constant is RG invariant inside give space-time sheet. The discrete p-adic coupling constant evolution replacing in TGD framework the ordinary RG evolution allows also formulation at space-time level.

The quantum phases $q = exp(i\pi/n)$ characterizing Jones inclusions define p-adic phase resolution in terms of the algebraic extension of p-adic numbers needed to represent q. The evolution of $\hbar$ is naturally associated with the p-adic phase resolution and allows a beautiful topological formulation at space-time level. This however requires a further generalization of the notion of imbedding space. This generalization is natural if one accepts that space-time and imbedding space emerge from an infinite-dimensional Clifford algebra extended to local algebra in a manner which is unique and from which TGD emerges.

The understanding of the quantization of Planck constants in terms of Jones inclusions has non-trivial implications concerning the understanding of the coupling constant evolution at quantitative level. One non-trivial implication is that one can in fact assume Kähler coupling strength to be RG invariant in p-adic coupling constant, an attractive idea that I had been forced to give up. The observations about general properties of induced classical color gauge fields combined with quantum classical correspondence inspire a hypothesis allowing to reduce the evolution of color coupling strength α_s to that of electro-weak U(1) coupling strength $\alpha_{U(1)}$, a result which is of considerable practical value.

The chapter consists of two parts. In the first sections quantitative predictions, which I dare to take rather seriously, are discussed. After that a general formulation of coupling constant evolution at space-time level and related interpretational issues are considered.

5.8 Does TGD Allow Quantum Field Theory Limits?

This chapter contains besides genuinely new material also old material which has become more or less obsolete with the advances occurred in the construction of quantum TGD described in the chapters "Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD" and "Construction of Quantum Theory". The conclusions from this work are following.

1. Quantum TGD can be formulated as a quantum field theory using modified Dirac action. This formulation is formally a free field theory for fermions and thus free of divergence difficulties. It satisfies the quantum gravitational holography principle and relies heavily on super-canonical and super Kac-Moody super algebras associated with light like 7-D and 3-D causal determinants which allow a promising matter to understand the classical non-determinism of Kähler action.

2. A completely new element is 7–3 duality and closely related effective 2-dimensionality, basically due to the conformal invariance related to the light like 3-D causal determinants. Effective 2-dimensionality leads to Feynman rules differing dramatically from those of perturbative quantum field theories. The particles on the internal lines are not labelled by four-momenta, and can be said to be on mass shell in the sense that the spinors in the propagator lines correspond to generalized eigen modes of the modified Dirac operator whereas the solutions of the modified Dirac equation represent $N = 4$ gauge super symmetries. Vertices are associated with 7-D CDs. Loop summations are absent and diagrams with loops are equivalent with tree diagrams. Also the finiteness of the S-matrix is manifest. Coupling constant evolution is replaced by a discrete coupling constant evolution induced by the infinite hierarchy of p-adic length scales each giving rise to a value of Kähler coupling strength analogous to a critical temperature.

The beauty and elegance of this approach makes the whole idea of perturbative QFT limit more or less obsolete, and it would seem that the notion of field theory limit should be replaced with a truncation of the full theory by posing restrictions on the energies of the interacting particles. p-Adic length scale hierarchy defines in a very natural manner this kind of truncation hierarchy.

The construction of the various limits of TGD should be based on following observations.

1. One can construct a whole family of limits. Both real and p-adic limits are possible and latter might provide an extremely simple and effective calculational tool. The classification of the particles according to the dimension D of the CP_2 projection of the particle like 3-surface implies that there are several limits. The most interesting $D = 4$ case corresponds to the identification of CP_2 type extremals as fundamental particles, whereas $D < 4$ extremals having at least one large spatial dimension, most probably serve as templates for topological condensation of CP_2 type extremals.

2. The limit of TGD should involve the effective elimination of configuration space degrees of freedom and their description phenomenologically. In particular, quark color is treated as spin like quantum numbers. Various coupling constants and propagators are inputs of limit and are predicted by the S-matrix constructed using the new Feynman rules. The 2-D surfaces X^2 identified as the intersections of light like 3-D and 7-D causal determinants are TGD counterparts of partons, and by the effective 2-dimensionality code all relevant data about quantum TGD. These surfaces are idealized as point like objects: partons become quarks and gluons. What is lost in this process is the possibility to describe bound states of partons from the first principles: QCD provides a basic example about this.

3. The only elegant manner to define the limits of TGD are as theories in Minkowski space. The information about the geometry of space-time surface is included implicitly in the process giving the coupling constants.

4. The notion of propagator in M^4 makes sense as an approximate concept if S-matrix elements can be expressed as tree diagrams using Feynman rules of effective field theory in M^4. The construction of the scalar propagator as a partition function in super-canonical algebra suggests that the propagator might have a universal form.

For completeness also the older approach based on the Yang-Mills Dirac (YMD) action for induced gauge fields modified by adding a quantum part and defined for maximum of Kähler function (absolute minimum of Kähler action) is discussed. This involves a generalization of the induction procedure so that it applies also the quantum fields. Propagators are put in by hand and propagator poles correspond to the masses of the particles predicted by the p-adic mass calculations. The only sensible interpretation of YMD action is as effective action giving rise only to tree diagrams: this of course conforms with the new view about Feynman diagrams. In principle Poincare invariance produces no problems. The description of family replication phenomenon and quark color require somewhat tricky constructions.

5.9 Equivalence of Loop Diagrams with Tree Diagrams and Cancellation of Infinities in Quantum TGD

The great dream of a physicist believing in reductionism and TGD would be a formalism generalizing Feynman diagrams allowing any graduate student to compute the predictions of the theory. TGD has forced myself to give up naive reductionism but I believe that TGD allows generalization of Feynman diagram in such a manner that one gets rid of the infinities plaguing practically all existing theories. The purpose of this chapter is to develop general vision about how this might be achieved. The vision is based on generalization of mathematical structures discovered in the construction of topological quantum field theories (TQFT), conformal field theories (CQFT). In particular, the notions of Hopf algebras and quantum groups, and categories are central. The following gives a very concise summary of the basic ideas.

5.9.1 Feynman diagrams as generalized braid diagrams

The first key idea is that generalized Feynman diagrams with diagrams which are analogous to knot and link diagrams in the sense that diagrams involving loops are equivalent with tree diagrams. This would be a generalization of duality symmetry of string models.

TGD itself provides general arguments supporting same idea. The identification of absolute minimum of Kähler action as a four-dimensional Feynman diagram characterizing particle reaction means that there is only single Feynman diagram instead of functional integral over 4-surfaces: this diagram is expected to be minimal one. S-matrix element as a representation of a path defining continuation of configuration space spinor field between different sectors of it corresponding different 3-topologies leads also to the conclusion that all continuations and corresponding Feynman diagrams are equivalent. Universe as a compute metaphor idea allowing quite concrete realization by generalization of what is meant by space-time point leads to the view that generalized Feynman diagrams characterize equivalent computations.

5.9.2 Coupling constant evolution from infinite number of critical values of Kähler coupling strength

The basic objection that this vision does not allow to understand coupling constant evolution involving loops in an essential manner can be circumvented. Quantum criticality requires that Kähler coupling constant α_K is analogous to critical temperature (so that the loops for configuration space integration vanish). The hypothesis motivated by the enormous vacuum degeneracy of Kähler action is that α_K has an infinite number of possible values labelled by p-adic length scales and also probably also by the dimensions of effective tensor factors defined hierarchy of II_1 factors (so called Beraha numbers) as found already earlier. The dependence on p-adic length scale L_p corresponds to the usual renormalization group evolution whereas the latter dependence would correspond to angular resolution and finite-dimensional extensions of p-adic number fields R_p. Finite finite resolution and renormalization group evolution are forced by the algebraic continuation of rational number based physics to real and p-adic number fields since p-adic and real notions of distance between rational points differ dramatically.

5.9.3 R-matrices, complex numbers, quaternions, and octonions

A crucial observation is that physically equivalent R-matrices of 6-vertex models are labelled by points of CP_2 whereas maximal space of commuting R-matrices are labelled by points of 2-sphere S^2. CP_2 also labels maximal associative and thus quaternionic subspaces of 8-dimensional octonion space containing a preferred imaginary unit whereas S^2 labels the maximally commutative subspaces of quaternion space, which suggests that R-matrices and these structures correspond to each other.

Number theoretic vision leads to the notion of number theoretic compactification meaning that space-time surfaces could be regarded either as hyper-quaternionic, and thus maximal associative, 4-surfaces in M^8 regarded as the space of hyper-octonions or as surfaces in $M^4 \times CP_2$ (the imaginary units of hyper-octonions are multiplied with $\sqrt{-1}$ so that the number theoretical norm has Minkowski signature). No dynamics is involved and duality might be more appropriate term. Associativity constraint is an essential element of also Yang-Baxter equations.

These observations lead to a concrete proposal how how quantum classical correspondence is realized classically at space-time level. Each point of CP_2 corresponds to R-matrix and 3-surfaces can be identified as preferred sections of space-time surface by requiring that unitary R-matrices are commuting in the section (micro-causality) whereas commutativity fails for R-matrices corresponding to different values of time coordinate defined by this foliation.

5.9.4 Ordinary conformal symmetries act on the space of super-canonical conformal weights

TGD predicts two kinds of super-conformal symmetries.

1. Quaternion conformal symmetries define super Kac-Moody representations and are realized at light-like 3-surfaces appearing as boundaries of space-time sheets and boundaries between space-time regions with Euclidian and Minkowskian signature of metric. Conformal weights are half integer valued in this case.

2. Super-canonical conformal invariance acts at the level of imbedding space and corresponds to light-like 7-surfaces of form $X_l^3 \times CP_2 \subset M^4 \times CP_2$ believed to appear as causal determinants too. In this case conformal weights are complex numbers of form $\Delta = n/2 + iy$ and for physical states they are expected to reduce to the form $\Delta = 1/2 + iy$.

Quantum classical correspondence suggests that the complex conformal weights of super-canonical algebra generators have space-time counterparts. The proposal is that the weights are mapped to the points of geodesic sphere of CP_2 (and thus also of space-time surface) labelling also mutually commutating R-matrices. The map is completely analogous to the map of momenta of quantum particles to the points of celestial sphere. One can thus regard super-generators as conformal fields in space-time or complex plane in which conformal weights appear as punctures. The action of super-conformal algebra and braid group on these points realizing monodromies of conformal field theories induces by a pull-back a braid group action on the conformal labels of configuration space gamma matrices (super generators) and corresponding isometry generators.

The gamma matrices and isometry generators spanning the super-canonical algebra can be regarded as fields of a conformal field theory in the complex plane containing infinite number of punctures defined by the complex conformal weights. Quaternion conformal Kac Moody and Super Virasoro algebras act as symmetries of this theory and S-matrix of TGD should involve the n-point functions of this conformal field theory.

This picture also justifies the earlier proposal that configuration space Clifford algebra defined by the gamma matrices acting as super generators defines an infinite-dimensional von Neumann algebra possessing hierarchies of type II_1 factors having a close connection with non-trivial representations of braid group and quantum groups. The zeros of Riemann Zeta along the line $Re(s) = 1/2$ in the plane of conformal weights could be regarded a the infinite braid behind the von Neumann algebra. Contrary to the expectations, also trivial zeros seem to be important. Super-canonical algebra predicts that also the superposition for the imaginary parts of non-trivial zeros makes sense so that one would have an entire hierarchy of one-dimensional lattices depending on the number of non-trivial zeros included. The braids defined by sub-lattices defined by subsets of non-trivial zeros could be seen as completely integrable 1-dimensional spin chains leading to a hierarchy of quantum groups and braiding matrices naturally.

It seems that not only Riemann's zeta but also polyzetas could play a fundamental role in TGD Universe. The super-canonical conformal weights of interacting particles, in particular of those forming bound states, are expected to have "off mass shell" values. An attractive hypothesis is that they correspond to the zeros of Riemann's polyzetas. Interaction would allow quite concretely the realization of braiding operations dynamically.

5.9.5 Equivalence of loop diagrams with tree diagrams from the axioms of generalized ribbon category

The fourth idea is that Hopf algebra related structures and appropriately generalized ribbon categories could provide a concrete realization of this picture. Generalized Feynman diagrams which are identified as braid diagrams with strands running in both directions of time and containing besides braid operations also boxes representing algebra morphisms with more than one incoming and outgoing strands describing particle reactions (3-particle vertex should be enough). In particular, fusion of 2-particles and decay of particle to two would correspond to generalizations of algebra product μ and co-product Δ to morphisms of the category defined by the super-canonical algebras associated with 3-surfaces with various topologies and conformal structures. The basic axioms for this structure generalizing Hopf algebra axioms state that diagrams with self energy loops, vertex corrections, and box diagrams are equivalent with tree diagrams.

5.9.6 Quantum criticality and renormalization group invariance

Quantum criticality means that renormalization group acts like isometry group at a fixed point rather than acting like a gauge symmetry as in the standard quantum field theory context. Despite this difference it is possible to understand how Feynman graph expansion with vanishing loop corrections relates to generalized Feynman graphs and a nice connection with the Hopf- and Lie algebra structures assigned by Connes and Kreimer to Feynman graphs emerges. The condition that loop diagrams are equivalent with tree diagrams gives explicit equations which might fix completely also the p-adic length scale evolution of vertices. Quantum criticality in principle fixes completely the values of the masses and coupling constants as a function of p-adic length scale.

5.9.7 The spectrum of zeros of Zeta and quantum TGD

The question whether the imaginary parts of the Riemann Zeta are linearly independent (as assumed in the previous work) or not is of crucial physical significance. Linear independence implies that the spectrum of the super-canonical weights is essentially an infinite-dimensional lattice. Otherwise a more complex structure results.

The hypothesis that p^{iy} is an algebraic phase for every prime implies that p^{iy} is expressible as a product of a Pythagorean phase and a root of unity for every prime p. The numerical evidence supporting the translational invariance of the correlations for the spectrum of zeros together with p-adic considerations leads to the working hypothesis that for any prime p one can express the spectrum of zeros as the product of a subset of Pythagorean prime phases and of a fixed subset U of roots of unity. The spectrum of zeros could be expressed as a union over the translates of the same basic spectrum defined by the roots of unity translated by the phase angles associated with a subset of Pythagorean phases: this is consistent with what the spectral correlations strongly suggest. That decompositions defined by different primes p yield the same spectrum would mean a powerful number theoretical symmetry realizing p-adicities at the level of the spectrum of Zeta.

The model for the scalar propagator as a partition function for the super-canonical algebra supports this view. The approximation that the zeros are linearly independent implies for any subalgebra defined by a finite set of zeros a universal spectrum of singularities for real values of mass squared, and at the limit of entire algebra the propagator is not mathematically defined since the singularities are dense in real axis. Linear independence however shifts the poles to complex values of mass and genuine resonances result and the propagator is expected to be well-defined for the entire algebra. The theory predicts universal momentum and p-adic length scale dependence of propagators in this picture and the option based on super-canonical algebra predicts very simple and elegant expression for the propagator with all the desired properties.

5.9.8 Quantum field theory formulation of quantum TGD

The super-canonical generalization of a 2-dimensional conformal field theory seems to be indispensable for the construction of S-matrix at the fundamental level and defines the vertices as n-point functions of a conformal field theory in turn used to construct S-matrix as tree diagrams. Also a quantum field theory defined for 3-surface X^3 belonging to the light like 7-surfaces $X_l^3 \times CP_2$

defining causal determinants seems to make sense. The QFT in question is determined by the absolute minimum $X^4(X^3)$ associated with the maximum of Kähler function and is consistent with the loop corrections in configuration space. The restriction to X^3 realizes quantum holography and means that a minimum amount of data about $X^4(X^3)$ is needed.

The modified Dirac action for the induced spinor fields at the maximum of the Kähler function with induced spinor fields identified as Grassmann algebra valued fields provides the sought for action. The fermionic propagator is defined by the inverse of the modified Dirac operator. Bosonic kinetic term is Grassmann algebra valued and vanishes for $\psi = 0$ and contributes nothing to the perturbation series in accordance with the effective freezing of the configuration space degrees of freedom. Since the fermionic action is free action, a divergence free quantum field theory is in question. One could also see the action as a fixed point of the map sending action to effective action in accordance with the idea that loop corrections vanish. Different sub-algebras of the super-canonical algebra correspond naturally to various conformally inequivalent configuration space metrics and infinite-dimensional subalgebras correspond to the singular limits when the space-time surface becomes vacuum extremal so that the modified Dirac operator vanishes identically and super-canonical propagator becomes ill-defined.

5.10 Was von Neumann right after all?

The work with TGD inspired model for quantum computation led to the realization that von Neumann algebras, in particular hyper-finite factors of type II_1 could provide the mathematics needed to develop a more explicit view about the construction of S-matrix. This has turned out to be the case to the extend that a general master formula for S-matrix with interactions described as a deformation of ordinary tensor product to Connes tensor products emerges.

1. Some background

It has been for few years clear that TGD could emerge from the mere infinite-dimensionality of the Clifford algebra of infinite-dimensional "world of classical worlds" and from number theoretical vision in which classical number fields play a key role and determine imbedding space and space-time dimensions. This would fix completely the "world of classical worlds".

Infinite-dimensional Clifford algebra is a standard representation for von Neumann algebra known as a hyper-finite factor of type II_1. In TGD framework the infinite tensor power of C(8), Clifford algebra of 8-D space would be the natural representation of this algebra.

2. How to localize infinite-dimensional Clifford algebra?

The basic new idea is to make this algebra *local*: local Clifford algebra as a generalization of gamma field of string models.

1. Represent Minkowski coordinate of M^d as linear combination of gamma matrices of D-dimensional space. This is the first guess. One fascinating finding is that this notion can be quantized and classical M^d is genuine quantum M^d with coordinate values eigenvalues of quantal commuting Hermitian operators built from matrix elements. Euclidian space is not obtained in this manner. Minkowski signature is something quantal and the standard quantum group $Gl_(2,q)(C)$ with (non-Hermitian matrix elements) gives M^4.

2. Form power series of the M^d coordinate represented as linear combination of gamma matrices with coefficients in corresponding infinite-D Clifford algebra. You would get tensor product of two algebra.

3. There is however a problem: one cannot distinguish the tensor product from the original infinite-D Clifford algebra. $D = 8$ is however an exception! You can replace gammas in the expansion of M^8 coordinate by hyper-octonionic units which are non-associative (or

octonionic units in quantum complexified-octonionic case). Now you cannot anymore absorb the tensor factor to the Clifford algebra and you get genuine M^8-localized factor of type II_1. Everything is determined by infinite-dimensional gamma matrix fields analogous to conformal super fields with z replaced by hyperoctonion.

4. Octonionic non-associativity actually reproduces whole classical and quantum TGD: space-time surface must be associative sub-manifolds hence hyper-quaternionic surfaces of M^8. Representability as surfaces in $M^4 \times CP_2$ follows naturally, the notion of configuration space of 3-surfaces, etc....

3. Connes tensor product for free fields as a universal definition of interaction quantum field theory

This picture has profound implications. Consider first the construction of S-matrix.

1. A non-perturbative construction of S-matrix emerges. The deep principle is simple. The canonical outer automorphism for von Neumann algebras defines a natural candidate unitary transformation giving rise to propagator. This outer automorphism is trivial for II_1 factors meaning that all lines appearing in Feynman diagrams must be on mass shell states satisfying Super Virasoro conditions. You can allow all possible diagrams: all on mass shell loop corrections vanish by unitarity and what remains are diagrams with single N-vertex.

2. At 2-surface representing N-vertex space-time sheets representing generalized Bohr orbits of incoming and outgoing particles meet. This vertex involves von Neumann trace (finite!) of localized gamma matrices expressible in terms of fermionic oscillator operators and defining free fields satisfying Super Virasoro conditions.

3. For free fields ordinary tensor product would not give interacting theory. What makes S-matrix non-trivial is that *Connes tensor product* is used instead of the ordinary one. This tensor product is a universal description for interactions and we can forget perturbation theory! Interactions result as a deformation of tensor product. Unitarity of resulting S-matrix is unproven but I dare believe that it holds true.

4. The subfactor $\mathcal{N}$ defining the Connes tensor product has interpretation in terms of the interaction between experimenter and measured system and each interaction type defines its own Connes tensor product. Basically $\mathcal{N}$ represents the limitations of the experimenter. For instance, IR and UV cutoffs could be seen as primitive manners to describe what $\mathcal{N}$ describes much more elegantly. At the limit when $\mathcal{N}$ contains only single element, theory would become free field theory but this is ideal situation never achievable.

5. Large $\hbar$ phases provide good hopes of realizing topological quantum computation. There is an additional new element. For quantum spinors state function reduction cannot be performed unless quantum deformation parameter equals to $q = 1$. The reason is that the components of quantum spinor do not commute: it is however possible to measure the commuting operators representing moduli squared of the components giving the probabilities associated with 'true' and 'false'. The universal eigenvalue spectrum for probabilities does not in general contain (1,0) so that quantum qbits are inherently fuzzy. State function reduction would occur only after a transition to q=1 phase and decoherence is not a problem as long as it does not induce this transition.

5.10.1 Does TGD predict the spectrum of Planck constants?

The quantization of Planck constant has been the basic them of TGD for more than one and half years. The breakthrough became with the realization that standard type Jones inclusions lead to a detailed understanding of what is involved and predict very simple spectrum for Planck constants associated with M^4 and CP_2 degrees of freedom. This picture allows to understand also gravitational Planck constant and coupling constant evolution and leads also to the understanding of ADE correspondences (index $\beta \leq 4$ and $\beta = 4$) from the point of view of Jones inclusions.

1. Jones inclusions and quantization of Planck constant

Jones inclusions combined with simple anyonic arguments turned out to be the key to the unification of existing heuristic ideas about the quantization of Planck constant.

1. The new view allows to understand how and why Planck constant is quantized and gives an amazingly simple formula for the separate Planck constants assignable to M^4 and CP_2 and appearing as scaling constants of their metrics. This in terms of a mild generalizations of standard Jones inclusions. The emergence of imbedding space means only that the scaling of these metrics have spectrum: their is no landscape.

2. In ordinary phase Planck constants of M^4 and CP_2 are same and have their standard values. Large Planck constant phases correspond to situations in which a transition to a phase in which quantum groups occurs. These situations correspond to standard Jones inclusions in which Clifford algebra is replaced with a sub-algebra of its G-invariant elements. G is product $G_a \times G_b$ of subgroups of $SL(2,C)$ and $SU(2)_L \times \times U(1)$ which also acts as a subgroup of SU(3). Space-time sheets are $n(G_b)$-fold coverings of M^4 and $n(G_a)$-fold coverings of CP_2 generalizing the picture which has emerged already. An elementary study of these coverings fixes the values of scaling factors of M^4 and CP_2 Planck constants to orders of the maximal cyclic sub-groups. Mass spectrum is invariant under these scalings.

3. This predicts automatically arbitrarily large values of Planck constant and assigns the preferred values of Planck constant to quantum phases $q = exp(i\pi/n)$ expressible using only iterated square root operation: these correspond to polygons obtainable by compass and ruler construction. In particular, experimentally favored values of $\hbar$ in living matter correspond to these special values of Planck constant. This model reproduces also the other aspects of the general vision. The subgroups of $SL(2,C)$ in turn can give rise to rescaling of $SU(3)$ Planck constant. The most general situation can be described in terms of Jones inclusions for fixed point subalgebras of number theoretic Clifford algebras defined by $G_a \times G_b \subset SL(2,C) \times SU(2)$.

4. These inclusions (apart from those for which G_a contains infinite number of elements) are represented by ADE or extended ADE diagrams depending on the value of index. The group algebras of these groups give rise to additional degrees of freedom which make possible to construct the multiplets of the corresponding gauge groups. For $\beta \leq 4$ the gauge groups A_n, D_2n, E_6, E_8 are possible so that TGD seems to be able to mimick these gauge theories. For $\beta = 4$ all ADE Kac Moody groups are possible and again mimicry becomes possible: TGD would be kind of universal physics emulator but it would be anyonic dark matter which would perform this emulation.

2. The values of gravitational Planck constant

The understanding of large Planck constants led to the detailed interpretation of what is involved with the emergence of gigantic gravitational Planck constant. The detailed spectrum for Planck constants gives very strong constraints to the values of $\hbar_{gr} = GMm/v_0$ if ones assumes that favored values of Planck constant correspond to the Jones inclusions for which quantum phase corresponds to a simple algebraic number expressible in terms of square roots of rationals. These phases correspond to n-polygons with n equal to a product of power of two and Fermat primes, which are all different. The ratios of planetary masses obey the predictions with an accuracy of 10 percent and GMm/v_0 for Sun-Earth system is consistent with $v_0 = 2^{-11}$ if the fraction of visible matter of all matter is about 6 per cent in solar system to be compared with the accepted cosmological value of 4 per cent.

3. Large values of Planck constant and coupling constant evolution

$\hbar_{gr}$ can be interpreted as Planck constant associated with CP_2 degrees of freedom and its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom meaning that dark matter cosmology has lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

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3. Identification of gravitational Planck constant as CP_2 Planck constant

$\hbar_{gr}$ can be interpreted as Planck constant associated with CP_2 degrees of freedom and its huge value implies that also the von Neumann inclusions associated with M^4 degrees of freedom meaning that dark matter cosmology has quantal lattice like structure with lattice cell given by H_a/G, H_a the $a = constant$ hyperboloid of M^4_+ and G subgroup of SL(2,C). The quantization of cosmic redshifts provides support for this prediction.

4. Large values of Planck constant and coupling constant evolution

Kähler coupling constant is the only coupling parameter in TGD. The original great vision is that Kähler coupling constant is analogous to critical temperature and thus uniquely determined. Later I concluded that Kähler coupling strength could depend on the p-adic length scale. The reason was that the prediction for the gravitational coupling strength was otherwise non-sensible. This motivated the assumption that gravitational coupling is RG invariant in the p-adic sense.

The expression of the basic parameter $v_0 = 2^{-11}$ appearing in the formula of $\hbar_{gr} = GMm/v_0$ in terms of basic parameters of TGD leads to the unexpected conclusion that α_K in electron length scale can be identified as electro-weak $U(1)$ coupling strength $\alpha_{U(1)}$. This identification is what group theory suggests but I had given it up since the resulting evolution for gravitational coupling was $G \propto L_p^2$ and thus completely un-physical. However, if gravitational interactions are mediated by space-time sheets characterized by Mersenne prime, the situation changes completely since M_{127} is the largest non-super-astrophysical p-adic length scale.

The second key observation is that all classical gauge fields and gravitational field are expressible using only CP_2 coordinates and classical color action and U(1) action both reduce to Kähler action. Furthermore, electroweak group U(2) can be regarded as a subgroup of color SU(3) in a well-defined sense and color holonomy is abelian. Hence one expects a simple formula relating various coupling constants. Let us take α_K as a p-adic renormalization group invariant in strong sense that it does not depend on the p-adic length scale at all.

The relationship for the couplings must involve $\alpha_{U(1)}$, α_s and α_K. The formula $1/\alpha_{U(1)}+1/\alpha_s = 1/\alpha_K$ states that the sum of $U(1)$ and color actions equals to Kähler action and is consistent with the decrease of the color coupling and the increase of the U(1) coupling with energy and implies a common asymptotic value $2\alpha_K$ for both. The hypothesis is consistent with the known facts about color and electroweak evolution and predicts correctly the confinement length scale as p-adic length scale assignable to gluons. The hypothesis reduces the evolution of α_s to the calculable evolution of electro-weak couplings: the importance of this result is difficult to over-estimate.

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